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## Geometry of 3-Spaces with Spinor Structure

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A special approach to examine spinor structure of 3-space is proposed. It is based on the use of the concept of a spatial spinor defined through taking the square root of a real-valued 3-vector. Two sorts of spatial spinor according to  $P$ -orientation of an initial 3-space are introduced: properly vector or pseudo vector one. These spinors,  $\eta$  and  $\xi$ , turned out to be different functions of Cartesian coordinates. To have a spinor space model, you ought to use a doubling vector space  $\{ (x_1, x_2, x_3) \otimes (x_1, x_2, x_3)' \}$ . The main idea is to develop some mathematical technique to work with such extended models. Spinor fields  $\eta$  and  $\xi$ , given as functions of Cartesian coordinates  $x_i \oplus x'_i$ , do not obey Cauchy-Riemann analyticity condition with respect to complex variable  $(x_1 + ix_2) \oplus (x_1 + ix_2)'$ . Spinor functions are in one-to-one correspondence with coordinates  $x_i \oplus x'_i$  everywhere excluding the whole axis  $(0, 0, x_3) \oplus (0, 0, x_3)'$  where they have an exponential discontinuity. It is proposed to consider properties of spinor fields  $\xi(x_i \oplus x'_i)$  and  $\eta(x_i \oplus x'_i)$  in terms of continuity with respect to geometrical directions in the vicinity of every point. The mapping of spinor field  $\eta$  into  $\xi$  and inverse have been constructed. Two sorts of spatial spinors are examined with the use of curvilinear coordinates  $(y_1, y_2, y_3)$ : cylindrical parabolic, spherical and parabolic ones. Transition from vector to spinor models is achieved by doubling initial parameterizing domain  $G(y_1, y_2, y_3) \implies \tilde{G}(y_1, y_2, y_3)$  with new identification rules on the boundaries. Different spinor space models are built on explicitly different spinor fields  $\xi(y)$  and  $\eta(y)$ . Explicit form of the mapping spinor field  $\eta(y)$  of pseudo vector model into spinor  $\xi(y)$  of properly vector one is given, it contains explicitly complex conjugation.

## 1 Introduction

In the literature, the problem of the so-called spinor structure of physical space-time was extensively discussed [1-40]. There were considered both possible experimental tests and mathematical methods to describe such a structure (see also [41-44]).

The main idea of the present treatment is to elaborate certain approach to this problem in the frame of mathematical technique, simple and natural as much as possible, for physicists without

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refined knowledge in contemporary topology and geometry. In other words, the idea is to show that old and naive mathematical tools based on the use of explicit coordinate language, which is yet of the most significance in any experiments-oriented physics, might be quite sufficient to describe adequately subtleties and peculiarities associated with possible spinor structure of physical space-time.

For simplicity, this work is restricted to a "non-relativistic" spinor model only when 2-component spinors of the unitary group  $SU(2)$  are taken into consideration. Brief preliminary remarks should be given of the concept of spatial spinor – primary mathematical object associated with a "point of a spinor space". We will start with the well-known Cartan's classification of 2-spinors with respect to spinor  $P$ -reflection: namely, the simplest irreducible representations of the unitary extended group

$$\tilde{SU}(2) = \left\{ g \in SU(2) \oplus J = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, \det g = +1, \det J = -1 \right\} \quad (1)$$

are 2-component spinors of two types  $T_A$

$$\begin{aligned} T_1 : \quad & T_1(g) = g, \quad T_1(J) = +J, \\ T_2 : \quad & T_2(g) = g, \quad T_2(J) = -J. \end{aligned} \quad (2)$$

With this in mind, there are two ways to construct 3-vector (complex-valued in general) in terms of 2-spinors

$$\begin{aligned} 1. \quad & (\xi \otimes \xi^*) = a + a_j \sigma^j, \quad a = \sqrt{a_j a_j}, \\ 2. \quad & (\eta \otimes \eta) = (c_j + i b_j) \sigma^j. \end{aligned} \quad (3)$$

From (3) it follows that when spinor  $\eta$  is either of the type 1 or of the type 2, real-valued 3-vector  $a_j$  is a pseudo vector. In turn, when  $\xi$  is either of the type 1 or of the type 2, real-valued 3-vectors  $c_j$  and  $b_j$  are both proper vectors. Evidently, variant 1 provides us with possibility to build a spinor model for pseudo vector 3-space  $\tilde{\Pi}_3$ , whereas variant 2 leads to a spinor model of properly vector 3-space  $\tilde{E}_3$ . In other words, according to which way of taking the square root of three real numbers – components of a 3-vector  $x_i$ , one will arrive at two different spatial spinors

$$\xi \Longleftrightarrow a_j, \quad \eta \Longleftrightarrow c_j \text{ or } (b_j). \quad (4)$$

These spinors,  $\eta$  and  $\xi$  respectively, turned out to be different functions of Cartesian coordinates  $(x_1, x_2, x_3)$ . Evidently, to have in hand a spinor space model, you are to use in a sense a doubling vector space

$$\{ (x_1, x_2, x_3) \otimes (x_1, x_2, x_3)' \}. \quad (5)$$

The main idea is to develop some mathematical technique to work with such extended models. Spinor fields  $\eta$  and  $\xi$ , constructed as functions of Cartesian coordinates  $x_i \oplus x'_i$ , do not obey Cauchy-Riemann analyticity condition with respect to complex variable  $(x_1 + ix_2)$ . Spinor functions are in one-to-one correspondence with coordinates  $x_i \oplus x'_i$  everywhere excluding the whole axis  $(0, 0, x_3) \oplus (0, 0, x_3)'$  they have an exponential and discrete  $\pm$ -sign discontinuities.

After extending models to spinor ones only exponential discontinuity remains. It was proposed to consider properties of spinor fields in terms of continuity with respect to geometrical directions in the vicinity of the every point.

In addition, two sorts of spatial spinors depending on  $P$ -orientation are examined with the use of curvilinear coordinates  $(y_1, y_2, y_3)$ . Transition from vector to spinor models is achieved by doubling initial parameterizing domain:  $G(y_1, y_2, y_3) \implies \tilde{G}(y_1, y_2, y_3)$  with new identification rules on the boundary. Different spinor space models are built on explicitly different spinor fields  $\xi$  and  $\eta$ . Explicit form of the mapping spinor field  $\eta(y)$  of pseudo vector model into spinor  $\xi(y)$  of properly vector one is given, it contain explicitly complex conjugation. Three most commonly used coordinate systems – spherical, parabolic ones, and cylindrical parabolic – have been considered in detail.

## 2 Pseudo vector space $\Pi_3$ and its spatial spinor $\xi$

Let  $\xi$  be either a spinor of the first or second type, then a conjugate spinor  $\xi^*$  will be of the second or first type respectively. Combining them into a 2-rank spinor, we get a pseudo scalar  $a$  and pseudo vector  $a_j$

$$(\xi \otimes \xi^*) = a + a_j \sigma^j, \quad a^{(J)} = +a, \quad a_j^{(J)} = +a_j. \quad (6)$$

Involved quantities transform under  $SU(2)$  according to (the notation is used  $(\vec{n} \times)_{ij} = \epsilon_{ijk} n_k$ )

$$\begin{aligned} \xi' &= B(n)\xi, \quad B(n) = In_0 - i\sigma^j n_j, \\ a'_j &= 0_{jl}(n)a_l, \quad 0(n) = I + 2[n_0 \vec{n} \times + (\vec{n} \times)^2]. \end{aligned} \quad (7)$$

The task is to find an explicit form of  $a$  and  $a_j$  in terms of spinor components. With the notation

$$\xi = \begin{pmatrix} x \\ y \end{pmatrix}, \quad (\xi \otimes \xi^*) = \begin{pmatrix} xx^* & xy^* \\ yx^* & yy^* \end{pmatrix}, \quad (8)$$

we have

$$\begin{aligned} a_1 &= \frac{1}{2}(yx^* + xy^*), \quad a_2 = \frac{i}{2}(xy^* - x^*y), \\ a_3 &= \frac{1}{2}(xx^* - yy^*), \quad a = \frac{1}{2}(xx^* + yy^*). \end{aligned} \quad (9)$$

Observing identity  $\vec{a}^2 = \frac{1}{4}(xx^* + yy^*)^2$ , one concludes that the scalar  $a$  is a positive square root of  $\vec{a}^2$ :  $a = +\sqrt{\vec{a}^2}$ .

Needless to say that multiplying an initial spinor  $\xi$  by a phase factor  $e^{i\alpha}$  does not affect both  $a$  and  $a_j$ ; this peculiarity will find its corollary in finding a spinor  $\xi$  from a given vector  $a_j$ . Now we are ready to invert relations (9). To this end one should take  $\eta$  in a special form ( $N, M \in [0, \infty)$ )

$$\xi = \begin{pmatrix} Ne^{in} \\ Me^{im} \end{pmatrix}, \quad n, m \in [-\pi, +\pi]. \quad (10)$$

Substituting (10) into (9), one gets

$$\begin{aligned} a_1 \pm ia_2 &= NM e^{\pm i(m-n)} , \\ a_3 &= \frac{1}{2}(N^2 - M^2), \quad a = \frac{1}{2}(N^2 + M^2) . \end{aligned} \quad (11)$$

From (11) one can see that components  $a_1$  and  $a_2$  determine only the difference  $(m - n)$ . In turn,  $a_3$  and  $a$  will fix  $M$  and  $N$ :

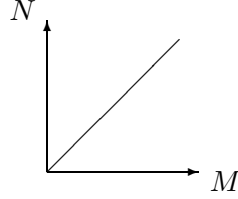


FIG.1.  $(M, N)$ -diagram

Here the line  $N = M$  corresponds to the plane  $a_3 = 0$ ; sub-set  $N > M$  refers to upper half-space  $\Pi_3^+(a_3 > 0)$ ; and sub-set  $N < M$  refers to lower half-space  $\Pi_3^-(a_3 < 0)$ ;  $M = 0$  refers to half-axis  $a_3 > 0$ ;  $N = 0$  refers to half-axis  $a_3 < 0$ ; initial point  $(0, 0, 0)$  is given by  $\xi = 0$ . For different regions of the  $\Pi$ -space the following designation will be used (see Fig. 2).

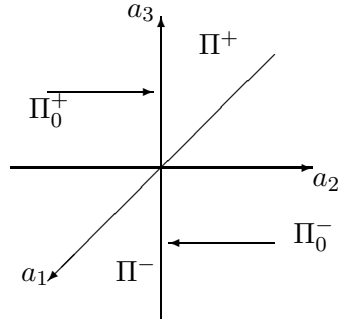


FIG. 2.  $\Pi$ -space regions

Instead of the variables  $n$  and  $m$  it is useful to take two new ones  $\gamma$  and  $\kappa$ :

$$\begin{aligned} \kappa &= (m + n) , \quad \gamma = (m - n) , \\ n &= \frac{1}{2}(\kappa - \gamma) , \quad m = \frac{1}{2}(\kappa + \gamma) . \end{aligned} \quad (12)$$

Correspondingly, the domain  $G(n, m)$ , a square centered in  $(0, 0)$  and with area  $(4\pi^2)$ , will change into a rhombus with area  $(8\pi^2)$ :

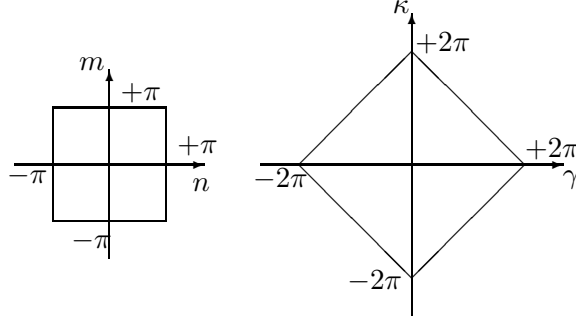


FIG. 3.  $(\gamma, k)$ -diagram

In the variables  $(\kappa; N, M, \gamma)$ , spinor  $\xi$  looks as (take note on a phase factor  $e^{ik/2}$ )

$$\xi = e^{i\kappa/2} \begin{pmatrix} N e^{-i\gamma/2} \\ M e^{+i\gamma/2} \end{pmatrix}, \quad (13)$$

and eqs. (11) will read

$$\begin{aligned} a_1 \pm ia_2 &= NM e^{\pm i\gamma}, \\ a_3 &= \frac{1}{2}(N^2 - M^2), \quad a = \frac{1}{2}(N^2 + M^2). \end{aligned} \quad (14)$$

One should note that the variable  $\kappa$  does not enter (14). Besides, ranging the variable  $\gamma$  in the interval  $[-2\pi, +2\pi]$  (see Fig. 3) ensures required double covering of the ordinary plane  $(a_1, a_2)$ . In other words, three parameters  $(M, N, \gamma)$  are sufficient to parameterize spinor model  $\tilde{\Pi}_3$  built upon a pseudo vector space  $\Pi_3$ . To this model  $\tilde{\Pi}_3$  there is a corresponded spinor field  $\xi(\vec{a})$  (in the following, the factor  $e^{ik/2}$  will be omitted)

$$\xi = \begin{pmatrix} \sqrt{a+a_3} e^{-i\gamma/2} \\ \sqrt{a-a_3} e^{+i\gamma/2} \end{pmatrix}, \quad e^{i\gamma} = \frac{a_1 + ia_2}{\sqrt{a_1^2 + a_2^2}}. \quad (15)$$

It should be noted that in describing  $\Pi_0^+$  and  $\Pi_0^-$  there arise peculiarities: at the whole axis  $a_3$  eqs. (15) contain ambiguity  $(0 + i0)/0$  (and expressions for  $\xi$  will contain a mute angle variable  $\Gamma : \gamma \rightarrow \Gamma$ )

$$\Pi_0^+ : \quad \xi_0^+ = \begin{pmatrix} \sqrt{+2a_3} e^{-i\Gamma/2} \\ 0 \end{pmatrix}, \quad (16)$$

$$\Pi_0^- : \quad \xi_0^- = \begin{pmatrix} 0 \\ \sqrt{-2a_3} e^{+i\Gamma/2} \end{pmatrix}, \quad (17)$$

$$e^{i\Gamma} = \lim_{a_1, a_2 \rightarrow 0} \frac{a_1 + ia_2}{\sqrt{a_1^2 + a_2^2}}. \quad (18)$$

At the plane  $a_3 = 0$ , spinor  $\xi$  reads as

$$\xi = \begin{pmatrix} \sqrt{a_1^2 + a_2^2} e^{-i\gamma/2} \\ \sqrt{a_1^2 + a_2^2} e^{+i\gamma/2} \end{pmatrix}. \quad (19)$$

### 3 Proper vector space $E_3$ and its spinor model $\tilde{E}_3$

In this Section we are going in the same line to define a spatial spinor associated with a proper vector space  $E_3$ . Let  $\eta$  be a spinor either of the first or second type. It leads to a 2-rank spinor  $(\eta \otimes \eta)$ , equivalent to a couple of real-valued proper vectors  $c_j$   $b_j$ :

$$(\eta \otimes \eta) = (c_j + i b_j) \sigma^j . \quad (20)$$

With respect to  $J$ -reflection, involved quantities  $\eta$  and  $(c_j, b_j)$  are transformed according to

$$\eta^{(J)} = +i\eta \text{ (or } -i\eta) , \quad c_j^{(J)} = -c_j , \quad b_j^{(J)} = -b_j , \quad (21)$$

and under continuous group  $SU(2)$

$$\eta' = B(n)\eta , \quad c'_i = 0_{ij}(n)c_j , \quad b'_i = 0_{ij}(n)b_j . \quad (22)$$

The task is to find vectors  $\vec{c}$  and  $\vec{b}$  in terms of spinor  $\eta$  components. With the same notation

$$\eta = \begin{pmatrix} Ne^{in} \\ Me^{im} \end{pmatrix} \quad (23)$$

after simple calculating we get

$$\begin{aligned} c_1 &= \frac{1}{2}(-M^2 \sin 2m + N^2 \sin 2n) , \\ c_2 &= \frac{1}{2}(+M^2 \cos 2m + N^2 \cos 2n) , \\ c_3 &= -MN \sin(m+n) , \\ b_1 &= \frac{1}{2}(M^2 \cos 2m - N^2 \cos 2n) , \\ b_2 &= \frac{1}{2}(M^2 \sin 2m + N^2 \sin 2n) , \\ b_3 &= +MN \cos(m+n) . \end{aligned} \quad (24)$$

Again, instead of  $(m, n)$  we will use  $(\kappa, \gamma)$  (see (12)), then eqs. (24) read as

$$\vec{c} = \vec{e}_f \cos \kappa - \vec{f} \sin \kappa , \quad \vec{b} = \vec{e}_f \sin \kappa + \vec{f} \cos \kappa , \quad (25)$$

where  $\vec{f}$  and  $\vec{e}_f$  are given by

$$\vec{f} = \begin{pmatrix} \frac{1}{2}(M^2 - N^2) \cos \gamma \\ \frac{1}{2}(M^2 - N^2) \sin \gamma \\ +MN \end{pmatrix} , \quad \vec{e}_f = \begin{pmatrix} -f \sin \gamma \\ +f \cos \gamma \\ 0 \end{pmatrix} .$$

All four vectors  $(\vec{f}, \vec{e}_f, \vec{c}, \vec{b})$  have the same length

$$|\vec{f}| = |\vec{e}_f| = |\vec{c}| = |\vec{b}| = (M^2 + N^2)/2 .$$

Besides, two orthogonality conditions  $\vec{f} \vec{e}_f = 0$  and  $\vec{b} \vec{c} = 0$  hold.

Now, we are at the point to determine certain sub-set of spinors in  $\eta(\kappa; N, M, \gamma)$ , which could be suitable to parameterize correctly spinor space  $\tilde{E}_3$ , covering twice an initial vector space  $E_3$ .

Starting from the set  $(\kappa = 0; N, M, \gamma)$  and respective sets of vectors  $\vec{c}$  and  $\vec{b}$ :

$$\vec{b} = +\vec{f}, \quad \vec{c} = +\vec{e}_f, \quad \vec{f} = \begin{pmatrix} \frac{1}{2}(M^2 - N^2) \cos \gamma \\ \frac{1}{2}(M^2 - N^2) \sin \gamma \\ +NN \end{pmatrix}, \quad (26)$$

and demanding parameters  $M, N, \gamma$  to be ranged as follows

$$M > N > 0, \quad \gamma \in [-2\pi, +2\pi]. \quad (27)$$

Vector  $\vec{b}$  covers upper half-space  $E_3^+$  twice; respective spinor  $\eta^+$  looks as

$$\eta^+ = \begin{pmatrix} \sqrt{b - (b_1^2 + b_2^2)^{1/2}} e^{-i\gamma/2} \\ \sqrt{b - (b_1^2 + b_2^2)^{1/2}} e^{+i\gamma/2} \end{pmatrix}, \quad e^{i\gamma} = \frac{b_1 + ib_2}{\sqrt{b_1^2 + b_2^2}}. \quad (28)$$

Now let us start with the sub-set  $(\kappa = \pi; N, M, \gamma)$  and respective vectors  $\vec{c}$  and  $\vec{b}$ :

$$\vec{b} = -\vec{f}, \quad \vec{c} = -\vec{e}_f, \quad \vec{f} = \begin{pmatrix} \frac{1}{2}(M^2 - N^2) \cos \gamma \\ \frac{1}{2}(M^2 - N^2) \sin \gamma \\ +NN \end{pmatrix}. \quad (29)$$

If one again expects the parameters  $M, N, \gamma$  to vary according to (27), then the vector  $\vec{b}$  in (29) covers a lower half-space  $E_3^-$  twice; expression for spinor  $\eta^-$  looks as

$$\eta^- = i \begin{pmatrix} \sqrt{b - (b_1^2 + b_2^2)^{1/2}} e^{-i\gamma/2} \\ \sqrt{b - (b_1^2 + b_2^2)^{1/2}} e^{+i\gamma/2} \end{pmatrix}, \quad e^{+i\gamma/2} = -i \sqrt{\frac{b_1 + ib_2}{\sqrt{b_1^2 + b_2^2}}}, \quad (30)$$

or in equivalent form

$$\eta^- = \begin{pmatrix} \sqrt{b - (b_1^2 + b_2^2)^{1/2}} \left[ -\sqrt{\frac{b_1 + ib_2}{(b_1^2 + b_2^2)^{1/2}}} \right]^* \\ \sqrt{b + (b_1^2 + b_2^2)^{1/2}} \left[ +\sqrt{\frac{b_1 + ib_2}{(b_1^2 + b_2^2)^{1/2}}} \right] \end{pmatrix}. \quad (31)$$

It is natural to expect a spinor field  $\eta$  to be continuous at the plane  $b_3 = 0$ , for this one must use in (30) and (28) the same square root of  $(b_1 + ib_2)$ . Thus, spinor  $\eta^{+\cap-}$  reads

$$\eta^{+\cap-} = \begin{pmatrix} 0 \\ \sqrt{2(b_1 + i b_2)} \end{pmatrix}. \quad (32)$$

Else one point should be clarified. Above, two variables  $m$  and  $n$  were taken as independent, each of them varies in the interval  $[-\pi, +\pi]$ . As a result, alternative variables  $(\gamma, \kappa)$  change inside the rhombus  $G(\gamma, \kappa)$  with area  $8\pi^2$  (see Fig. 3).

In accordance with this, the variable  $\gamma \in G(\gamma, \kappa = 0)$  will lie automatically inside the interval  $[-2\pi, +2\pi]$ . It is just we need to parameterize spinor half-space. In turn,  $\gamma \in G(\gamma, \kappa = \pi/2)$

lies only in the interval  $[-\pi, +\pi]$ . However, to parameterize spinor half-space we need that the variable  $\gamma \in [-2\pi, +2\pi]$ .

There is no contradiction here because the domain  $G(n, m)$  is equivalent to both the domain  $G(\kappa, \gamma)$  mentioned above and another domain  $G'(\gamma, \kappa)$  (identification rules of the boundary points see in diagrams below)

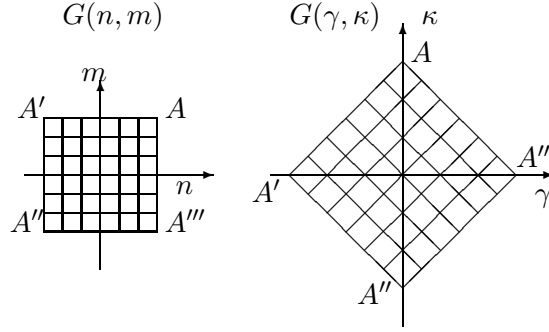


FIG.4.  $G(\gamma, \kappa)$ -diagram

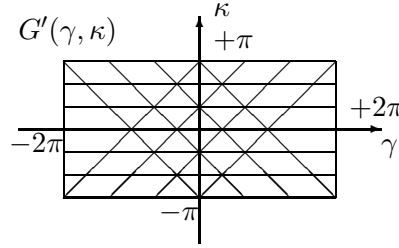


FIG. 5.  $G'(\gamma, \kappa)$ -diagram

Transition from  $G(\kappa, \gamma)$  to  $G'(\kappa, \gamma)$  can be additionally explained by the diagram

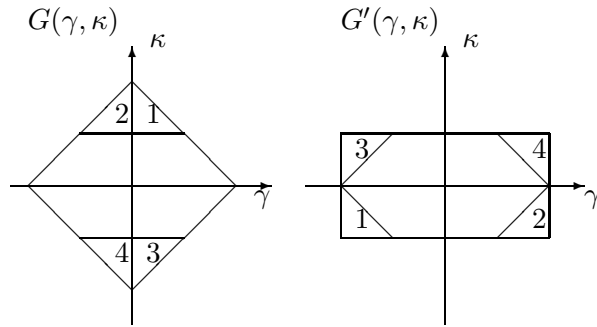


FIG. 6. Transition from  $G(\gamma, \kappa)$  to  $G'(\gamma, \kappa)$



In closing, let us dwell on peculiarities in parameterizing subsets  $\tilde{E}_0^+$  and  $\tilde{E}_0^-$  by spinor field  $\eta$ . As for  $\tilde{E}_0^+$  we have

$$\vec{b} = +\vec{f}, \quad \vec{c} = +\vec{e}_f, \quad \vec{f} = \begin{pmatrix} 0 \\ 0 \\ +MN \end{pmatrix}. \quad (33)$$

That is  $M = N$  and the variable  $\gamma$  is mute, therefore

$$\eta_0^{(+)} = \sqrt{+b_3} \begin{pmatrix} e^{-i\Gamma/2} \\ e^{+i\Gamma/2} \end{pmatrix}, \quad e^{i\Gamma} = \lim_{b_1 \rightarrow 0, b_2 \rightarrow 0} \frac{b_1 + ib_2}{\sqrt{b_1^2 + b_2^2}}. \quad (34)$$

Analogously, for  $\tilde{E}_0^+$  we have

$$\vec{b} = -\vec{f}, \quad \vec{c} = -\vec{e}_f, \quad \vec{f} = \begin{pmatrix} 0 \\ 0 \\ +MN \end{pmatrix}, \quad (35)$$

$$\eta_0^{(-)} = \sqrt{-b_3} \begin{pmatrix} -e^{-i\Gamma/2} \\ +e^{+i\Gamma/2} \end{pmatrix}, \quad e^{i\Gamma} = \lim_{b_1 \rightarrow 0, b_2 \rightarrow 0} \frac{b_1 + ib_2}{\sqrt{b_1^2 + b_2^2}}. \quad (36)$$

## 4 Spatial spinor $\xi_{a_3}(a_1 + ia_2)$ and Cauchy-Riemann analitycity

It is natural to consider two components of spinor field  $\xi = \xi(a_j)$  as complex-valued functions of  $z = a_1 + ia_2$  and a real-valued coordinate  $a_3$ :

$$\xi = \begin{pmatrix} \xi_{a_3}^1(a_1 + ia_2) \\ \xi_{a_3}^2(a_1 + ia_2) \end{pmatrix}. \quad (37)$$

Since spinor components depend upon  $a_1 + ia_2$  and its conjugate  $a_1 - ia_2$ , they do not differentiable in Cauchy-Riemann sense. Let us enlarge on the subject. Cauchy-Riemann (C-R) condition has the form

$$z = x + iy, \quad f(z) = U + iV, \quad \frac{\partial U}{\partial x} - \frac{\partial V}{\partial y} = 0, \quad \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} = 0. \quad (38)$$

For spinor components it will be convenient to use the notation

$$\begin{aligned} \xi^1 &= f^+(\cos \frac{\gamma}{2} - i \sin \frac{\gamma}{2}) = U^1 + iV^1, \\ \xi^2 &= f^-(\cos \frac{\gamma}{2} + i \sin \frac{\gamma}{2}) = U^2 + iV^2, \end{aligned} \quad (39)$$

where

$$f^\pm = \sqrt{a \pm a_3}, \quad e^{+i\gamma/2} = \cos \frac{\gamma}{2} + i \sin \frac{\gamma}{2}. \quad (40)$$

The formulas will be needed:

$$\frac{\partial f^+}{\partial a_1} = \frac{a_1}{2a\sqrt{a + a_3}}, \quad \frac{\partial f^+}{\partial a_2} = \frac{a_2}{2a\sqrt{a + a_3}},$$

$$\frac{\partial f^-}{\partial a_1} = \frac{a_1}{2a\sqrt{a-a_3}}, \quad \frac{\partial f^-}{\partial a_2} = \frac{a_2}{2a\sqrt{a-a_3}},$$

and

$$\begin{aligned} \frac{\partial}{\partial a_1} e^{\pm i\gamma/2} &= -e^{\pm i\gamma/2} \frac{\pm ia_2}{2\rho^2}, & \frac{\partial}{\partial a_2} e^{\pm i\gamma/2} &= +e^{\pm i\gamma/2} \frac{\pm ia_1}{2\rho^2}, \\ \frac{\partial}{\partial a_1} \cos \frac{\gamma}{2} &= +\frac{a_2}{2\rho^2} \sin \frac{\gamma}{2}, & \frac{\partial}{\partial a_2} \cos \frac{\gamma}{2} &= -\frac{a_1}{2\rho^2} \sin \frac{\gamma}{2}, \\ \frac{\partial}{\partial a_1} \sin \frac{\gamma}{2} &= -\frac{a_2}{2\rho^2} \cos \frac{\gamma}{2}, & \frac{\partial}{\partial a_2} \sin \frac{\gamma}{2} &= +\frac{a_1}{2\rho^2} \cos \frac{\gamma}{2}; \end{aligned}$$

where  $\rho = \sqrt{a_1^2 + a_2^2}$ .

Derivatives  $\partial U^1/\partial a_j$  and  $\partial V^1/\partial a_j$  are

$$\begin{aligned} \frac{\partial U^1}{\partial a_1} &= \cos \frac{\gamma}{2} \frac{a_1}{2a\sqrt{a+a_3}} + \sin \frac{\gamma}{2} \frac{a_2\sqrt{a+a_3}}{2\rho^2}, \\ \frac{\partial U^1}{\partial a_2} &= \cos \frac{\gamma}{2} \frac{a_2}{2a\sqrt{a+a_3}} - \sin \frac{\gamma}{2} \frac{a_1\sqrt{a+a_3}}{2\rho^2}, \\ \frac{\partial V^1}{\partial a_1} &= -\sin \frac{\gamma}{2} \frac{a_1}{2a\sqrt{a+a_3}} + \cos \frac{\gamma}{2} \frac{a_2\sqrt{a+a_3}}{2\rho^2}, \\ \frac{\partial V^1}{\partial a_2} &= -\sin \frac{\gamma}{2} \frac{a_2}{2a\sqrt{a+a_3}} - \cos \frac{\gamma}{2} \frac{a_1\sqrt{a+a_3}}{2\rho^2}; \end{aligned}$$

and derivatives  $\partial U^2/\partial a_j$ ,  $\partial V^2/\partial a_j$  are

$$\begin{aligned} \frac{\partial U^2}{\partial a_1} &= \cos \frac{\gamma}{2} \frac{a_1}{2a\sqrt{a-a_3}} + \sin \frac{\gamma}{2} \frac{a_2\sqrt{a-a_3}}{2\rho^2}, \\ \frac{\partial U^2}{\partial a_2} &= \cos \frac{\gamma}{2} \frac{a_2}{2a\sqrt{a-a_3}} - \sin \frac{\gamma}{2} \frac{a_1\sqrt{a-a_3}}{2\rho^2}, \\ \frac{\partial V^2}{\partial a_1} &= \sin \frac{\gamma}{2} \frac{a_1}{2a\sqrt{a-a_3}} - \cos \frac{\gamma}{2} \frac{a_2\sqrt{a-a_3}}{2\rho^2}, \\ \frac{\partial V^2}{\partial a_2} &= \sin \frac{\gamma}{2} \frac{a_2}{2a\sqrt{a-a_3}} + \cos \frac{\gamma}{2} \frac{a_1\sqrt{a-a_3}}{2\rho^2}. \end{aligned}$$

With the use of these equations, we arrive at modified Cauchy-Riemann relations

$$\begin{aligned} \frac{\partial U^1}{\partial a_1} - \frac{\partial V^1}{\partial a_2} &= \frac{1}{2} \left( a_1 \cos \frac{\gamma}{2} + a_2 \sin \frac{\gamma}{2} \right) \left[ \frac{1}{a\sqrt{a+a_3}} + \frac{\sqrt{a+a_3}}{\rho^2} \right], \\ \frac{\partial U^1}{\partial a_2} + \frac{\partial V^1}{\partial a_1} &= \frac{1}{2} \left( a_2 \cos \frac{\gamma}{2} - a_1 \sin \frac{\gamma}{2} \right) \left[ \frac{1}{a\sqrt{a+a_3}} + \frac{\sqrt{a+a_3}}{\rho^2} \right], \\ \frac{\partial U^2}{\partial a_1} - \frac{\partial V^2}{\partial a_2} &= \frac{1}{2} \left( a_1 \cos \frac{\gamma}{2} - a_2 \sin \frac{\gamma}{2} \right) \left[ \frac{1}{a\sqrt{a-a_3}} - \frac{\sqrt{a-a_3}}{\rho^2} \right], \\ \frac{\partial U^2}{\partial a_2} + \frac{\partial V^2}{\partial a_1} &= \frac{1}{2} \left( a_2 \cos \frac{\gamma}{2} + a_1 \sin \frac{\gamma}{2} \right) \left[ \frac{1}{a\sqrt{a-a_3}} - \frac{\sqrt{a-a_3}}{\rho^2} \right]. \end{aligned}$$

If  $a_3 = 0$ , we get

$$\begin{aligned}\frac{\partial U^1}{\partial a_1} - \frac{\partial V^1}{\partial a_2} &= \frac{1}{\sqrt{\rho}} \cos \frac{\gamma}{2}, \\ \frac{\partial U^1}{\partial a_2} + \frac{\partial V^1}{\partial a_1} &= \frac{1}{\sqrt{\rho}} \sin \frac{\gamma}{2}, \\ \frac{\partial U^2}{\partial a_1} - \frac{\partial V^2}{\partial a_2} &= 0, \quad \frac{\partial U^2}{\partial a_2} + \frac{\partial V^2}{\partial a_1} = 0\end{aligned}$$

which is quite understandable if we take into account the form of spinor  $\xi$  at  $a_3 = 0$

$$\xi^{+\cap-} = \begin{pmatrix} (\sqrt{a_1 + ia_2})^* \\ \sqrt{a_1 + ia_2} \end{pmatrix}. \quad (41)$$

At  $\rho \rightarrow \infty$  C-R condition will hold.

Special note should be given to behavior of the spinor field  $\xi^i$  along half-plane  $\{a_1 \geq 0, a_2 = 0\}^{a_3}$ . Here spinor  $\xi$  is not a single-valued function of spatial points of the vector space  $Pi_3$  because its values depend on direction from which one approaches the points.

## 5 Calculating $\nabla \xi$ and $\nabla_{\vec{n}} \xi$

Spatial spinor field  $\xi^{a_3}(a_1 + ia_2)$  is not differentiable in the C-R sense. However, some continuity property of the spinor field yet exists. With this in mind, let us calculate 2-gradient of  $\xi(a_j)$ :

$$\nabla \xi = \left( \frac{\partial}{\partial a_1} \xi, \frac{\partial}{\partial a_2} \xi \right), \quad \xi = \xi^{a_3}(a_1, a_2). \quad (42)$$

This quantity could serve as characteristics of smoothness of spinor field  $(\xi^1, \xi^2)$ . With the use of formulas from previous Section one readily gets

$$\begin{aligned}\frac{\partial}{\partial a_1} \xi^1 &= \frac{1}{2} \xi^1 \left( \frac{a_1}{a(a + a_3)} + i \frac{a_2}{\rho^2} \right), \\ \frac{\partial}{\partial a_2} \xi^1 &= \frac{1}{2} \xi^1 \left( \frac{a_2}{a(a + a_3)} - i \frac{a_1}{\rho^2} \right),\end{aligned} \quad (43)$$

$$\begin{aligned}\frac{\partial}{\partial a_1} \xi^2 &= \frac{1}{2} \xi^2 \left( \frac{a_1}{a(a + a_3)} - i \frac{a_2}{\rho^2} \right), \\ \frac{\partial}{\partial a_2} \xi^2 &= \frac{1}{2} \xi^2 \left( \frac{a_2}{a(a + a_3)} - i \frac{a_1}{\rho^2} \right).\end{aligned} \quad (44)$$

The form of these equations will look shorter if one uses gradient along directions  $\nabla_{\vec{n}} \xi = (\vec{n} \nabla \xi)$  in the vicinity of every point. From (43) and (44) it follows

$$\begin{aligned}\nabla_{\vec{n}} \xi^1 &= \frac{1}{2} \left[ \frac{(\vec{n} \vec{a})}{a(a + a_3)} + i \frac{\vec{n} \times \vec{a}}{\rho^2} \right] \xi^1, \\ \nabla_{\vec{n}} \xi^2 &= \frac{1}{2} \left[ \frac{(\vec{n} \vec{a})}{a(a - a_3)} - i \frac{\vec{n} \times \vec{a}}{\rho^2} \right] \xi^2,\end{aligned} \quad (45)$$

where

$$(\vec{n} \vec{a}) = n_1 a_1 + n_2 a_2, \quad \vec{n} \times \vec{a} = n_1 a_2 - n_2 a_1.$$

For every vector  $\vec{a} = (a_1, a_2)$  one can consider two directions  $\vec{n}$ , parallel and orthogonal to it. If  $\vec{n} = \vec{n}_{\parallel}$ , then  $(\vec{n} \vec{a}) = 0$  and

$$\nabla_{\parallel} \xi^1 = \frac{1}{2} \frac{(\vec{n} \vec{a})}{a(a + a_3)} \xi, \quad \nabla_{\parallel} \xi^2 = \frac{1}{2} \frac{(\vec{n} \vec{a})}{a(a - a_3)} \xi. \quad (46)$$

If  $\vec{n} = \vec{n}_{\perp}$  then  $(\vec{n} \vec{a}) = 0$  and

$$\nabla_{\perp} \xi^1 = \frac{i}{2} \frac{\vec{n} \times \vec{a}}{\rho^2} \xi^1, \quad \nabla_{\perp} \xi^2 = -\frac{i}{2} \frac{\vec{n} \times \vec{a}}{\rho^2} \xi^2. \quad (47)$$

In other words, the equations have the structure

$$\nabla_{\vec{n}} \xi = \nabla_{\parallel} \xi + \nabla_{\perp} \xi.$$

The relations (45) can be re-written in matrix form

$$\nabla_{\vec{n}} \xi = A \xi. \quad (48)$$

Relation (48) can be considered alternatively as a master equation that prescribes the explicit form of spinor  $\xi(\vec{a})$  – from which we had started in the beginning. This estimation of equation (48) seems to be interesting and possibly fruitful. As for now, it does not look simple or fundamental indeed, however having been in their infancy it does have exiting mathematical potential.

## 6 Spinor field $\eta$ peculiarities

In this Section we are going to examine more closely singular properties of spinor field  $\xi^{a_3}(a_1, a_2)$ . At this, three cases,  $a_3 < 0$ ,  $a_3 = 0$ ,  $a_3 > 0$  should be considered separately.

Evidently, there exist peculiarities on the whole axis  $(0, 0, a_3)$  and along the whole half-plane  $(a_1 \geq 0, a_2 = 0)^{a_3}$ . For every point of the axis, instead of a single value, spinor has a set of values (mute variable  $\Gamma$ ). At every point of the half-plane, instead of a single value, spinor has two ones, different in sign – assuming the vector space model is investigated in terms of spinor field. Therefore, the quantity  $\nabla_{\vec{n}} \xi$  cannot be calculated without trouble in these peculiar sets  $\{\vec{a}^0\}$  – where spinor  $\xi$  losses single-valuedness. As an alternative, for these points there may be determined another characteristics

$$\nabla_{\vec{n}}^{\vec{m}} \xi(\vec{a}^0) = \lim_{\epsilon \rightarrow 0} \nabla_{\vec{n}} \xi(\vec{a}^0 + \epsilon \vec{m}), \quad (49)$$

that is one should find the quantity  $\nabla_{\vec{n}} \xi$  in the vicinity of singular point  $\vec{a}^0$  and then passes to the limit approaching to  $\vec{a}^0$  along different ways. In this line, let us consider the neighborhood of  $(0, 0)$  at  $\Pi_3^+$ :

$$\vec{a} = \vec{a}^0 + \epsilon \vec{m}, \quad \vec{a}^0 = (0, 0), \quad \epsilon \rightarrow 0.$$

Taking  $\epsilon$  as a small parameter we get to  $\tilde{\Pi}_0^+$  :

$$e^{+i\gamma} \sim (m_1 + im_2) = e^{+iM} , \quad (a - a_3) \sim \frac{\epsilon^2}{2a_3} ,$$

$$\xi^1 \sim \sqrt{2a_3} e^{-iM/2} , \quad \xi^2 \sim \frac{\epsilon}{\sqrt{2a_3}} e^{+iM/2} .$$

Substituting these into (49), we arrive at

$$\tilde{\Pi}_0^+ , \quad \nabla_{\vec{n}}^{\vec{m}} \xi^1(0,0) = \frac{1}{2} \sqrt{2a_3} e^{-iM/2} \left[ \epsilon \frac{(\vec{n} \cdot \vec{m})}{2a_3^2} + i \frac{(\vec{n} \times \vec{m})}{\epsilon} \right] , \quad (50)$$

$$\nabla_{\vec{n}}^{\vec{m}} \xi^2(0,0) = \frac{e^{+iM/2}}{2\sqrt{2a_3}} \left[ (\vec{n} \cdot \vec{m}) - i \frac{\vec{n} \times \vec{m}}{2} \right] . \quad (51)$$

Here the vector  $\vec{m}$  cannot be taken as  $\vec{m}_0 = (1, 0)$  – because, if it is so, the vector  $\vec{a} = (\vec{a}^0 + \epsilon \vec{m}_0)$  will get into a singular set where  $\nabla_{\vec{n}} \xi$  is not well defined. Instead, one should analyze two limits only:

$$\lim_{\vec{m} \rightarrow \vec{m}_0^+} \nabla_{\vec{n}}^{\vec{m}} \xi^i(0,0) = - \lim_{\vec{m} \rightarrow \vec{m}_0^-} \nabla_{\vec{n}}^{\vec{m}} \xi^i(0,0) . \quad (52)$$

Designation  $\vec{m} \rightarrow \vec{m}_0^+$  means that  $\vec{m}$  approaches to  $\vec{m}_0$  from up half-plane, whereas  $\vec{m} \rightarrow \vec{m}_0^-$  assumes that  $\vec{m}$  approaches to  $\vec{m}_0$  from lower half-plane.

In the same way, consideration of the neighborhood of  $(0,0)$  in  $\Pi_3^-$  leads to

$$\tilde{\Pi}_0^- : \nabla_{\vec{n}}^{\vec{m}} \xi^1(0,0) = \frac{e^{-iM/2}}{\sqrt{-2a_3}} \left[ \vec{n} \cdot \vec{m} + \frac{i}{2} \vec{n} \times \vec{m} \right] , \quad (53)$$

$$\nabla_{\vec{n}}^{\vec{m}} \xi^2(0,0) = \frac{1}{2} \sqrt{-2a_3} e^{+iM/2} \left[ \epsilon \frac{\vec{n} \cdot \vec{m}}{2a_3^2} - i \frac{\vec{n} \times \vec{m}}{\epsilon} \right] . \quad (54)$$

As for the point  $\tilde{\Pi}_0^{+\cap-}$  we will have

$$\xi^1 \sim \sqrt{\epsilon} e^{-iM/2} , \quad \xi^2 \sim \sqrt{\epsilon} e^{+iM/2} \quad (55)$$

and further

$$\nabla_{\vec{n}}^{\vec{m}} \xi_0^{+\cap-} = \frac{1}{2\sqrt{\epsilon}} \begin{pmatrix} e^{-iM/2} [\vec{n} \cdot \vec{m} + i \vec{n} \times \vec{m}] \\ e^{+iM/2} [\vec{n} \cdot \vec{m} - i \vec{n} \times \vec{m}] \end{pmatrix} . \quad (56)$$

In a sense, for every plane  $(a_2, a_2)^{a_3}$  its infinite boundary is peculiar as well — expression for  $\nabla_{\vec{n}} \xi$  at the line  $\{\infty m_1, \infty m_2\}^{a_3}$  will be  $(\Omega \rightarrow \infty)$

$$\nabla_{\vec{n}}^{\vec{m}} \xi(\infty) = \frac{1}{2\sqrt{\Omega}} \begin{pmatrix} e^{-iM/2} [\vec{n} \cdot \vec{m} + i \vec{n} \times \vec{m}] \\ e^{+iM/2} [\vec{n} \cdot \vec{m} - i \vec{n} \times \vec{m}] \end{pmatrix} . \quad (57)$$

Now, is is the point to examine spinor peculiarities at the half-plane  $\{a_1 > 0, a_2 = 0\}^{a_3}$ :

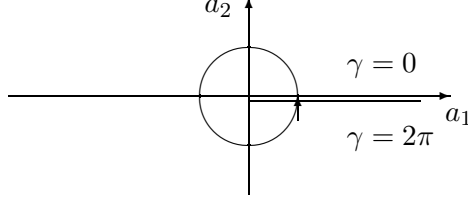


FIG. 7. Spinor peculiarities at  $a_2 = 0$  :  $\xi(\gamma = 0) = -\xi(\gamma = 2\pi)$

Here spinor field is double-valued. To describe that behavior let us act in the way used above:

$$\lim_{\epsilon \rightarrow 0} \nabla_{\vec{n}} \xi(\vec{a}^0 + \epsilon \vec{m}) = \nabla_{\vec{n}}^{\vec{m}} \xi(\vec{a}^0) ,$$

$$\vec{a}^0 = (a_1^0 > 0, a_2^0 = 0) , \quad \vec{m} \neq \pm \vec{m}_0 = \pm(1, 0) .$$

Taking into consideration

$$a_1 \sim a_1^0 + \epsilon m_1 , \quad a_2 \sim \epsilon m_2 ,$$

$$\vec{n} \cdot \vec{a} \sim +n_1 a_1^0 + \epsilon \vec{n} \cdot \vec{m} , \quad \vec{n} \times \vec{a} \sim -n_2 a_1^0 + \epsilon \vec{n} \times \vec{m} ,$$

and

$$\lim_{\epsilon \rightarrow 0} \xi^1(\vec{a}_0 + \epsilon \vec{m}) = \sqrt{a^0 + a_3^0} \operatorname{sgn}(m_2) ,$$

$$\lim_{\epsilon \rightarrow 0} \xi^2(\vec{a}_0 + \epsilon \vec{m}) = \sqrt{a^0 - a_3^0} \operatorname{sgn}(m_2) ,$$

we easily obtain

$$\nabla_{\vec{n}}^{\vec{m}} \xi^1 = \frac{a_1^0}{2\sqrt{a^0 + a_3^0}} \operatorname{sgn}(m_2) \left( \frac{n_1}{a^0} - i \frac{n_2}{a^0 - a_3^0} \right) ,$$

$$\nabla_{\vec{n}}^{\vec{m}} \xi^2 = \frac{a_1^0}{2\sqrt{a^0 - a_3^0}} \operatorname{sgn}(m_2) \left( \frac{n_1}{a^0} + i \frac{n_2}{a^0 + a_3^0} \right) .$$

These relations can be accompanied by the diagram

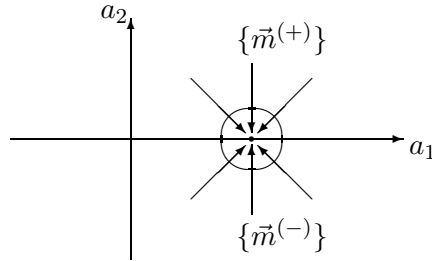


FIG. 8. Spinor peculiarities and  $\pi$ -vicinities

That is one may isolate two angular  $\pi$ -vicinities near the point  $\vec{a}_0$  — within each of them there is no dependence on  $\vec{m}$ , but

$$\nabla_{\vec{n}}^{\vec{m}^{(-)}} \xi = -\nabla_{\vec{n}}^{\vec{m}^{(+)}} \xi . \quad (58)$$

Now, one can make some general remarks on the mapping  $\Pi_3 \Rightarrow \xi$ ,  $\tilde{\Pi}_3 \Rightarrow \xi$  over the vector  $\Pi_3$  and spinor  $\tilde{\Pi}_3$  space models. The mapping  $\Pi_3 \Rightarrow \xi$  may be illustrated by the diagrams

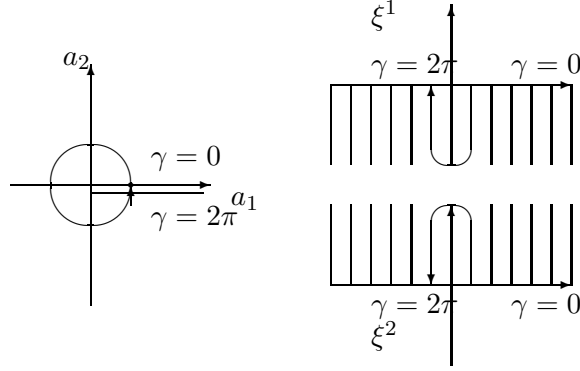


FIG. 9. Spinor discontinuity

that is the whole real plane  $(a_1, a_2)$  maps into a couple of complex half-planes  $\xi^1$  and  $\xi^2$ , differently oriented. For these maps the existence of discontinuity along a positive half-axis

$$\Pi_3, a_2 = 0, a_1 \geq 0 : \quad \xi(\gamma = 0) = -\xi(\gamma = 2\pi)$$

is inevitable. In contrast to this, the mapping  $\tilde{\Pi}_3 \Rightarrow \xi$  looks more smooth:

$$\tilde{\Pi}_3, a_2 = 0, a_1 \geq 0 : \quad \xi(\gamma = -2\pi) = +\xi(\gamma = +2\pi).$$

In other words, changing vector model into spinor one may be considered as a way to ensure continuity property of spinor field  $\xi$  in maximally large domain. In this context, the use of 2-sheeted planes instead of 1-sheeted planes appears to be natural and intelligible operation.

Initial vector space  $\Pi_3$  could be thought of as a collection of all 1-sheeted  $a_3$ -planes:

$$\Pi_3 = \sum_{a_3 \in (-\infty, +\infty)} (a_1, a_2)^{a_3} ,$$

instead an extended space  $\tilde{\Pi}_3$ , one may imagine spinor one as a collection of all 2-sheeted  $a_3$ -planes:

$$\tilde{\Pi}_3 = \sum_{a_3 \in (-\infty, +\infty)}^{\delta=1,2} (a_1, a_2)^{a_3} .$$

Any 2-sheeted plane differs in topological sense from 1-sheeted – now neighborhood of a zero point  $(0, 0)$  is not Euclidean. Therefore, extended space  $\tilde{\Pi}_3$  will be non-Euclidean as well. The

concept of nearness in such a model should take special attention to: nearness in Euclidean sense  $\Pi_3$  is not the same as the nearness for extended model  $\tilde{\Pi}_3$ . Indeed, two points can be near to each other only if they both belong to the same sheet or if they approach to a sewing domain. For example, the following points

$$\{a_1^{\delta=1}, a_2^{\delta=1}\}_{a'_3}, \{a_1^{\delta=1}, a_2^{\delta=1}\}_{a''_3}, \text{ if } (a''_3 - a'_3) \rightarrow 0$$

are neighboring ones; analogously close will be the points (if  $(a''_3 - a'_3) \rightarrow 0$ )

$$\{a_1^{\delta=2}, a_2^{\delta=2}\}_{a'_3}, \quad \{a_1^{\delta=2}, a_2^{\delta=2}\}_{3''_3}.$$

However, two points  $\{a_1^{\delta=1}, a_2^{\delta=1}\}_{a'_3}$  and  $\{a_1^{\delta=2}, a_2^{\delta=2}\}_{a'_3}$  will be quite distant from each other if they do not belong to a sewing domain.

In a precise form, changing space  $\Pi_3$  into extended space  $\tilde{\Pi}_3$  results in

for model  $\Pi_3$

- 1) spinor  $\xi(\vec{a})$  is exponentially discontinuous at the points  $(0,0)_{a_3}$  and  $(\pm)$ -valued along half-plane  $(0, a_2 = 0)_{a_3}$ ;
- 2) spinor  $\xi = \xi(\vec{a}, \vec{m})$  has discontinuity on a unique direction near to  $(0,0)_{a_3}$  and on two direction near the half-plane  $(0, a_2 = 0)_{a_3}$ .

for model  $\tilde{\Pi}_3$

- 1) spinor  $\xi(\vec{a}^{\delta=1,2})$  is exponentially discontinued at the points  $(0,0)_{a_3}$ ; any points of  $(\pm)$ -valued discontinuity does not exist;
- 2) spinor  $\xi(\vec{a}^{\delta=1,2}, \vec{m})$  is continuous everywhere.

So, the change of a space model  $\Pi_3$  substantially alters underlined spinor field's continuity properties. In the next sections, in the same line, we are going to examine spinor geometry of properly vector space  $E_3$ . It seems important, in a parallel way to have both spinor models, resulting respectively from different  $P$ -orientations of an initial space. The main idea is to make explicit manifestations of geometrical difference of pseudo and properly vector space models apparent as much as possible.

## 7 Spinor $\eta^{b_3}(b_1 + ib_2)$ and analyticity

Let us consider spinor components  $\eta(b_1, b_2, b_3)$  as complex-valued functions of  $z = (b_1 + ib_2)$  and parameter  $b_3$  (let  $\sigma = \pm 1$ ):

$$\eta^{(\sigma)}(b_j) = \begin{pmatrix} \eta^{1(\sigma)}(b_3, b_1 + ib_2) \\ \eta^{2(\sigma)}(b_3, b_1 + ib_2) \end{pmatrix}. \quad (59)$$



The notation will be used

$$\begin{aligned}\eta^{1(\sigma)} &= \sigma g^-(\cos \frac{\gamma}{2} - i \sin \frac{\gamma}{2}) = U^{1(\sigma)} + iV^{1(\sigma)}, \\ \eta^2 &= g^+(\cos \frac{\gamma}{2} + i \sin \frac{\gamma}{2}) = U^2 + iV^2, \\ g^\pm &= \sqrt{b \pm (b_1^2 + b_2^2)}, \\ e^{i\gamma/2} &= \sqrt{\frac{b_1 + ib_2}{(b_1^2 + b_2^2)^{1/2}}} = \cos \frac{\gamma}{2} + i \sin \frac{\gamma}{2}.\end{aligned}$$

Derivatives will be needed

$$\begin{aligned}\frac{\partial}{\partial b_1} g^\pm &= \pm \frac{b_1}{b} \frac{\sqrt{b \pm (b_1^2 + b_2^2)^{1/2}}}{2\sqrt{b_1^2 + b_2^2}}, \quad \frac{\partial}{\partial b_2} g^\pm = \pm \frac{b_2}{b} \frac{\sqrt{b \pm (b_1^2 + b_2^2)^{1/2}}}{2\sqrt{b_1^2 + b_2^2}}, \\ \frac{\partial}{\partial b_1} e^{\pm i\gamma/2} &= -e^{\pm i\gamma/2} \frac{\pm ib_2}{2\rho^2}, \quad \frac{\partial}{\partial b_2} e^{\pm i\gamma/2} = -e^{\pm i\gamma/2} \frac{\pm ib_1}{2\rho^2}, \\ \frac{\partial}{\partial b_2} \cos \frac{\gamma}{2} &= -\frac{b_1}{2\rho^2} \sin \frac{\gamma}{2}, \quad \frac{\partial}{\partial b_2} \sin \frac{\gamma}{2} = +\frac{b_1}{2\rho^2} \cos \frac{\gamma}{2},\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial b_1} U^{1(\sigma)} &= \sigma \frac{\sqrt{b-\rho}}{2\rho} \left[ -\frac{b_1}{b} \cos \frac{\gamma}{2} + \frac{b_2}{\rho} \sin \frac{\gamma}{2} \right], \\ \frac{\partial}{\partial b_2} U^{1(\sigma)} &= \sigma \frac{\sqrt{b-\rho}}{2\rho} \left[ -\frac{b_2}{b} \cos \frac{\gamma}{2} - \frac{b_1}{\rho} \sin \frac{\gamma}{2} \right], \\ \frac{\partial}{\partial b_1} V^{1(\sigma)} &= \sigma \frac{\sqrt{b-\rho}}{2\rho} \left[ -\frac{b_1}{b} \sin \frac{\gamma}{2} + \frac{b_2}{\rho} \cos \frac{\gamma}{2} \right], \\ \frac{\partial}{\partial b_2} V^{1(\sigma)} &= \sigma \frac{\sqrt{b-\rho}}{2\rho} \left[ +\frac{b_2}{b} \sin \frac{\gamma}{2} - \frac{b_1}{\rho} \cos \frac{\gamma}{2} \right];\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial b_1} U^2 &= \frac{\sqrt{b+\rho}}{2\rho} \left[ +\frac{b_1}{b} \cos \frac{\gamma}{2} + \frac{b_2}{\rho} \sin \frac{\gamma}{2} \right], \\ \frac{\partial}{\partial b_2} U^2 &= \frac{\sqrt{b+\rho}}{2\rho} \left[ +\frac{b_2}{b} \cos \frac{\gamma}{2} - \frac{b_1}{\rho} \sin \frac{\gamma}{2} \right], \\ \frac{\partial}{\partial b_1} V^2 &= \frac{\sqrt{b+\rho}}{2\rho} \left[ +\frac{b_1}{b} \sin \frac{\gamma}{2} - \frac{b_2}{\rho} \cos \frac{\gamma}{2} \right], \\ \frac{\partial}{\partial b_2} V^2 &= \frac{\sqrt{b+\rho}}{2\rho} \left[ +\frac{b_2}{b} \sin \frac{\gamma}{2} + \frac{b_1}{\rho} \cos \frac{\gamma}{2} \right].\end{aligned}$$

Again, we find the modified Cauchy-Riemann relations

$$\begin{aligned}\frac{\partial}{\partial b_1} U^{1(\sigma)} - \frac{\partial}{\partial b_2} V^{1(\sigma)} &= \sigma \frac{\sqrt{b-\rho}}{2\rho} \left( \frac{1}{\rho} - \frac{1}{b} \right) \left[ b_1 \cos \frac{\gamma}{2} + b_2 \sin \frac{\gamma}{2} \right], \\ \frac{\partial}{\partial b_2} U^{1(\sigma)} + \frac{\partial}{\partial b_1} V^{1(\sigma)} &= \sigma \frac{\sqrt{b-\rho}}{2\rho} \left( \frac{1}{\rho} - \frac{1}{b} \right) \left[ b_1 \cos \frac{\gamma}{2} - b_2 \sin \frac{\gamma}{2} \right],\end{aligned}\tag{60}$$

$$\begin{aligned}\frac{\partial}{\partial b_1}U^2 - \frac{\partial}{\partial b_2}V^2 &= \frac{\sqrt{b+\rho}}{2\rho}\left(\frac{1}{\rho} - \frac{1}{b}\right)\left[-b_1 \cos \frac{\gamma}{2} + b_2 \sin \frac{\gamma}{2}\right], \\ \frac{\partial}{\partial b_2}U^2 + \frac{\partial}{\partial b_1}V^2 &= \frac{\sqrt{b+\rho}}{2\rho}\left(\frac{1}{\rho} - \frac{1}{b}\right)\left[-b_1 \cos \frac{\gamma}{2} - b_2 \sin \frac{\gamma}{2}\right].\end{aligned}\quad (61)$$

When  $b_3 = 0$ , from (60) and (61) it follows

$$\begin{aligned}\frac{\partial}{\partial b_1}U^{1(\sigma)} - \frac{\partial}{\partial b_2}V^{1(\sigma)} &= 0, & \frac{\partial}{\partial b_2}U^{1(\sigma)} + \frac{\partial}{\partial b_1}V^{1(\sigma)} &= 0, \\ \frac{\partial}{\partial b_1}U^2 - \frac{\partial}{\partial b_2}V^2 &= 0, & \frac{\partial}{\partial b_2}U^2 - \frac{\partial}{\partial b_1}V^2 &= 0,\end{aligned}$$

that is C-R relations hold. It is consistent with the form of spinor  $\eta$  at  $b_3 = 0$ :

$$\eta^{+\cap-} = \sqrt{2\rho} \begin{pmatrix} 0 \\ e^{+i\gamma/2} \end{pmatrix} = \sqrt{2} \begin{pmatrix} 0 \\ \sqrt{b_1 + ib_2} \end{pmatrix}.$$

## 8 Spinor $\eta$ continuity properties

The 2-gradient of spinor field  $\eta$  will be (symbol  $\sigma$  at  $\eta^1$  is omitted):

$$\begin{aligned}\frac{\partial}{\partial b_1}\eta^1 &= \eta^1 \frac{1}{2\rho} \left(-\frac{b_1}{b} + i\frac{b_2}{b}\right), & \frac{\partial}{\partial b_2}\eta^1 &= \eta^1 \frac{1}{2\rho} \left(-\frac{b_2}{b} - i\frac{b_1}{b}\right), \\ \frac{\partial}{\partial b_1}\eta^2 &= \eta^2 \frac{1}{2\rho} \left(+\frac{b_1}{b} - i\frac{b_2}{b}\right), & \frac{\partial}{\partial b_2}\eta^2 &= \eta^2 \frac{1}{2\rho} \left(+\frac{b_2}{b} + i\frac{b_1}{b}\right).\end{aligned}\quad (62)$$

From (62) it follows

$$\nabla_{\vec{n}}\eta^1 = \eta^1 \frac{1}{2\rho} \left[-\frac{1}{b}(\vec{n} \cdot \vec{b}) + \frac{i}{\rho}(\vec{n} \times \vec{b})\right], \quad \nabla_{\vec{n}}\eta^2 = \eta^2 \frac{1}{2\rho} \left[+\frac{1}{b}(\vec{n} \cdot \vec{b}) - \frac{i}{\rho}(\vec{n} \times \vec{b})\right]. \quad (63)$$

Here again (see in Section 6) one can see two terms:

$$\nabla_{\vec{n}}\eta = (\nabla_{\perp}\eta + \nabla_{\parallel}\eta).$$

In the case  $b_3 = 0$  relations (63) look much simpler

$$\eta_{b_3=0}^1 = 0, \quad \nabla_{\vec{n}}\eta_{b_3=0}^2 = \eta_{b_3=0}^2 \frac{1}{2\rho^2} [\vec{n} \cdot \vec{b} - i \vec{n} \times \vec{b}]. \quad (64)$$

## 9 Peculiarities of field $\eta^{b_3}(b_1 + ib_2)$

Consideration of the problem will be performed in the manner used in Section 6. In the neighborhood of  $(0,0)^{b_3} \in \hat{E}_3^+$  there is

$$\begin{aligned}\vec{b} &\sim (\epsilon m_1, \epsilon m_2, b_3), \quad b_3 > 0, \\ \eta^1 &\sim +\sqrt{b_3} e^{-iM/2}, \quad \eta^2 \sim +\sqrt{b_3} e^{+iM/2}, \\ \nabla_{\vec{n}}^{\vec{m}}\eta^1 &\sim \frac{\sqrt{b_3}e^{-iM/2}}{2} \left[-\frac{(\vec{n}\vec{m})}{b_3} + i\frac{(\vec{n} \times \vec{m})}{\epsilon}\right], \\ \nabla_{\vec{n}}^{\vec{m}}\eta^2 &\sim \frac{\sqrt{b_3}e^{+iM/2}}{2} \left[+\frac{(\vec{n}\vec{m})}{b_3} - \frac{(\vec{n} \times \vec{m})}{\epsilon}\right],\end{aligned}\quad (65)$$

here  $\vec{m} \neq (+1, 0, 0)$ . Analogously, for  $(0, 0)^{b_3} \in \tilde{E}_3^-$  there is

$$\begin{aligned}\vec{b} &\sim (\epsilon m_1, \epsilon m_2, b_3), \quad b_3 < 0, \\ \eta^1 &\sim -\sqrt{-b_3}e^{-iM/2}, \quad \eta^2 \sim +\sqrt{-b_3}e^{+iM/2}, \\ \nabla_{\vec{n}}^{\vec{m}} \eta^1 &\sim \frac{\sqrt{-b_3}e^{-iM/2}}{2} \left[ +\frac{(\vec{n}\vec{m})}{|b_3|} - i\frac{(\vec{n} \times \vec{m})}{\epsilon} \right], \\ \nabla_{\vec{n}}^{\vec{m}} \eta^2 &\sim \frac{\sqrt{-b_3}e^{+iM/2}}{2} \left[ +\frac{(\vec{n}\vec{m})}{|b_3|} - \frac{(\vec{n} \times \vec{m})}{\epsilon} \right].\end{aligned}$$

Near the points  $E_0^{+\cap-}$ , when  $\vec{b} \sim (\epsilon m_1, \epsilon m_2, 0)$ , we have

$$\begin{aligned}\eta^1 &= 0, \quad \eta^2 = \sqrt{2\epsilon}e^{+iM/2}, \quad \nabla_{\vec{n}}^{\vec{m}} \eta^1 = 0, \\ \nabla_{\vec{n}}^{\vec{m}} \eta^2 &= \frac{e^{+iM/2}}{2\epsilon} [\vec{n} \vec{m} - i \vec{n} \times \vec{m}].\end{aligned}\tag{66}$$

For half-axis  $\{b_1^0 > 0, b_2^0 = 0\}$  we will have (the notation  $b^0 = \sqrt{(b_1^0)^2 + (b_3^0)^2}$  is used)

$$\begin{aligned}\nabla_{\vec{n}}^{\vec{m}} \eta^{1(\sigma)} &= \sigma \frac{\sqrt{b^0 - b_1^0}}{2} \left[ -\frac{n_1}{b_0} - i\frac{n_2}{b_1^0} \right] \text{sgn}(m_2), \\ \nabla_{\vec{n}}^{\vec{m}} \eta^2 &= \sigma \frac{\sqrt{b^0 + b_1^0}}{2} \left[ +\frac{n_1}{b_0} + \frac{n_2}{b_1^0} \right] \text{sgn}(m_2).\end{aligned}$$

Everything said in the end of Section 5 on the pseudo vector model is applied here too; it is unnecessary to repeat the same else one time.

## 10 Comparing models $\xi$ and $\eta$

Now we are going to describe some qualitative distinctions between spinor models  $\xi$  and  $\eta$ . Two models of spinors spaces with respect to  $P$ -orientation are grounded on different mappings  $\xi$  and  $\eta$  defined over the same extended domain  $\tilde{G}(y_i)$ . The natural question is: how are these two maps connected to each others. An answer can be found on comparing the formulas for  $\xi$  and  $\eta$ . An answer can be straightforwardly found. Indeed, taking into account identities

$$\begin{aligned}\frac{1}{\sqrt{2}}(\sqrt{x+x_3} + \sqrt{x-x_3}) &= +\sqrt{x+\rho}, \quad x_3 > 0, \\ \frac{1}{\sqrt{2}}(\sqrt{x+x_3} - \sqrt{x-x_3}) &= -\sqrt{x-\rho}, \quad x_3 < 0, \\ \frac{1}{\sqrt{2}}(\sqrt{x+x_3} + \sqrt{x-x_3}) &= +\sqrt{x+\rho},\end{aligned}$$

one can straightforwardly arrive at

$$\eta_1 = \frac{\xi_1 - \xi_2^*}{\sqrt{2}}, \quad \eta_2 = \frac{\xi_1^* + \xi_2}{\sqrt{2}}$$

or in more short form

$$\eta = \frac{1}{\sqrt{2}}(\xi - i \sigma^2 \xi^*) . \quad (67)$$

Inverse to (67) looks as

$$\xi_1 = \frac{\eta_1 + \eta_2^*}{\sqrt{2}} , \quad \xi_2 = \frac{\eta_2 - \eta_1^*}{\sqrt{2}} ,$$

or

$$\xi = \frac{1}{\sqrt{2}} (\eta - i \sigma^2 \eta^*) . \quad (68)$$

In connection with eqs. (67) and (68) there are two points to which special attention must be given:

1) complex conjugation enters them explicitly which correlates with the change in orientation properties of the models;

2) spinors  $\xi$  and  $i\sigma^2 \xi^*$  (as well as  $\eta$  and  $i\sigma^2 \eta^*$ ) provide us with non-equivalent representations of the extended unitary group  $\tilde{SU}(2)$ .

We have seen that description of differently  $P$ -oriented geometries in terms of spinor fields  $\eta$  and  $\xi$  has made hardly noticeable distinction between these two geometries much more apparent and intuitively appreciable as connected with different types of spatial geometry indeed.

## 11 Spinors $\xi$ and $\eta$ in cylindrical parabolic coordinates

This coordinate system in initial  $E_3$ -space is defined by the relations

$$x_1 = \frac{y_1^2 - y_2^2}{2} , \quad x_2 = y_1 y_2 , \quad x_3 = y_3 , \quad y_2 \in [0, +\infty) , \quad y_1, y_3 \in (-\infty, +\infty) . \quad (69)$$

They can be illustrated by the figure

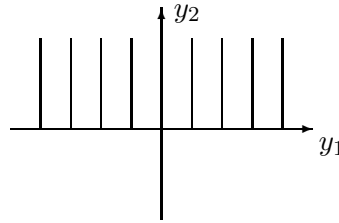


FIG. 10. Region  $G(y_1, y_2)^{y_3}$

where domain  $G(y_1, y_2)^{y_3}$  (at arbitrary  $y_3$ ) ranging in the half-plane  $(y_1, y_2)$  covers the whole vector plane  $(x_1, x_2)^{x_3}$ .

The spinor  $\xi$  of pseudo vector  $\Pi_3$ -model is given by

$$\xi(y) = \begin{pmatrix} \sqrt{(y_3^2 + (y_1^2 + y_2^2)^2/4)^{1/2} + y_3} e^{-i\gamma/2} \\ \sqrt{(y_3^2 + (y_1^2 + y_2^2)^2/4)^{1/2} - y_3} e^{+i\gamma/2} \end{pmatrix} , \quad e^{i\gamma/2} = \frac{y_1 + iy_2}{\sqrt{y_1^2 + y_2^2}} , \quad (70)$$

where the factor  $e^{i\gamma/2}$  runs through upper complex half-plane. The one-to-one correspondence  $\xi \longleftrightarrow (y_1, y_2, y_3)$  is violated at  $x_3$ -axis, at these peculiar point sets  $\Pi_0^+$  and  $\Pi_0^-$  spinor looks as

$$\xi_0^+ = \sqrt{+2y_3} \begin{pmatrix} e^{-i\Gamma/2} \\ 0 \end{pmatrix}, \quad \xi_0^- = \sqrt{-2y_3} \begin{pmatrix} 0 \\ e^{+i\Gamma/2} \end{pmatrix}, \quad (71)$$

where a mute angle variable  $\Gamma$  is used

$$e^{+i\Gamma/2} = \lim_{y_1 \rightarrow 0, y_2 \rightarrow 0} \frac{y_1 + iy_2}{\sqrt{y_1^2 + y_2^2}}.$$

In the plane  $\Pi^{+\cap-}$  spinor  $\xi$  is given by

$$\xi^{+\cap-} = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1 - iy_2 \\ y_1 + iy_2 \end{pmatrix}. \quad (72)$$

For a proper vector model, formulas for  $\eta$ -spinor look as (values  $+$  and  $-$  taken by symbol  $\sigma$  there correspond to  $x_3 > 0$  and  $x_3 < 0$  half-spaces respectively)

$$\eta^\sigma(y) = \begin{pmatrix} \sqrt{\sqrt{y_3^2 + (y_1^2 + y_2^2)^2/4} - \frac{y_1^2 + y_2^2}{2}} \sigma e^{-i\gamma/2} \\ \sqrt{\sqrt{y_3^2 + (y_1^2 + y_2^2)^2/4} + \frac{y_1^2 + y_2^2}{2}} e^{-i\gamma/2} \end{pmatrix}. \quad (73)$$

Now we are to extend the vector  $E_3$  and  $\Pi_3$  models to spinor ones. To this end it is convenient to employ two new variables  $k$  and  $\phi$  instead of  $y_1, y_2$ :

$$y_1 = k \cos \phi, \quad y_2 = k \sin \phi, \quad \phi \in [0, \pi];$$

in  $x$ -representation we get to

$$x_1 = \frac{k^2}{2} \cos 2\phi, \quad x_2 = \frac{k^2}{2} \sin 2\phi, \quad 2\phi \in [0, 2\pi]$$

that leads to the following identification rule in the set of boundary points of the domain  $G(y_1, y_2)^{y_3}$  (covering vector spaces  $\Pi_3$  and  $E_3$ ):

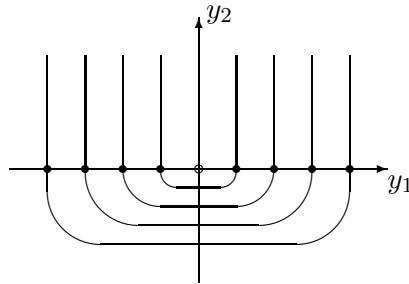


FIG. 11. Region  $G(y_1, y_2)^{y_3}$

here identified points on the boundary are connected by lines.

Bearing in mind that spinors  $\xi(y)$  and  $\eta(y)$  take on different, opposite in sign, values one can put forward the following simple way to construct extended (spinor) models  $\tilde{E}_3$  and  $\tilde{\Pi}_3$ : it is sufficient to double the range of  $y_2$ -variable:

$$y_2 \in [0, +\infty) \implies y_2 \in (-\infty, +\infty).$$

After so doing the above factor  $e^{+i\gamma/2}$  will run through the full unit circle:

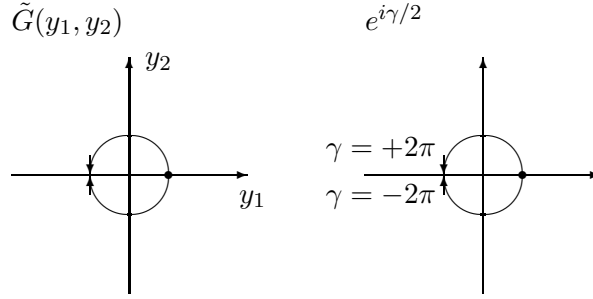


FIG. 12.  $4\pi$  - continuity

It is important to note the substantial changing in the identification rules at the boundary set of  $G(y_1, y_2, y_3)$  — now for extended domain  $\tilde{G}(y_1, y_2, y_3)$  one needs no special rules at all. Thus, in a sense, the domain  $\tilde{G}(y_1, y_2, y_3)$  appears to be simpler than  $G(y_1, y_2, y_3)$ .

Else one point must be emphasized. The same extended set  $\tilde{G}(y_1, y_2, y_3)$  is valid to both spinor models  $\xi(y)$  and  $\eta(y)$ . This means that only choice of the set with doubling dimension and identification rules does not determine in full the whole geometry of spinor spaces. Specification of their  $P$ -orientation requires seemingly additional information about this set. Unfortunately, this point has not been clarified sufficiently. Searching the model under consideration for some arguments to state those distinctions in rational way is the main objective of the present work.

Evidently,  $P$ -orientation manifests itself in explicitly different spinor functions  $\xi(y)$  and  $\eta(y)$ . Some qualitative distinction between these spinor functions is revealed if one follows orientation of spinor  $(\xi_1, \xi_2)$  and  $(\eta_1, \eta_2)$  while going from  $x_3^+$  - half-space to  $x_3^-$  - half-space. Here the explaining diagrams may be given:

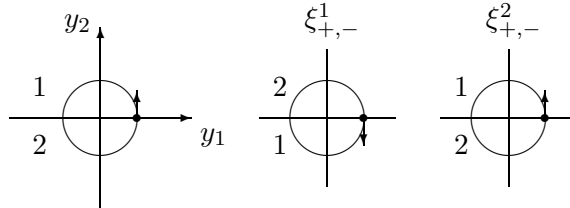


FIG. 13. ( $\xi$  - model)

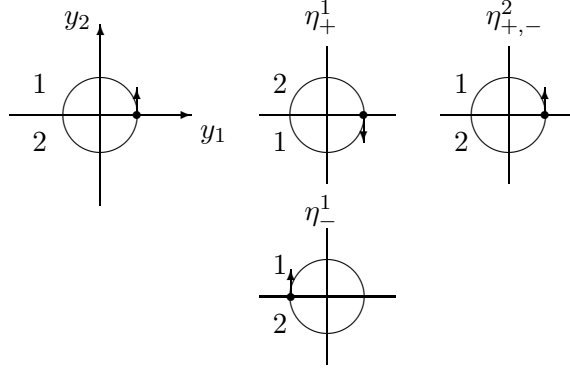


FIG. 14. (  $\eta$  - model)

Numbers 1 and 2 there correspond to first (initial) and second (additional) sub-space of the whole space with spinor structure.

Else one method to describe spatial spinor  $\xi(y)$  and  $\eta(y)$  with the help of coordinates  $y_i$  is the 2-gradient:

$$\begin{aligned} \frac{\partial}{\partial y_1} \xi^1 &= \frac{\xi^1}{2} \left( \frac{\rho}{a(a+a_3)} y_1 + \frac{i}{\rho} y_2 \right), & \frac{\partial}{\partial y_2} \xi^1 &= \frac{\xi^1}{2} \left( \frac{\rho}{a(a+a_3)} y_2 - \frac{i}{\rho} y_1 \right), \\ \frac{\partial}{\partial y_1} \xi^2 &= \frac{\xi^2}{2} \left( \frac{\rho}{a(a-a_3)} y_1 - \frac{i}{\rho} y_2 \right), & \frac{\partial}{\partial y_2} \xi^2 &= \frac{\xi^2}{2} \left( \frac{\rho}{a(a-a_3)} y_2 + \frac{i}{\rho} y_1 \right), \end{aligned} \quad (74)$$

$$\begin{aligned} \frac{\partial}{\partial y_1} \eta^1 &= \frac{\eta^1}{2} \left( -\frac{y_1}{b} + i \frac{y_2}{\rho} \right), & \frac{\partial}{\partial y_2} \eta^1 &= \frac{\eta^1}{2} \left( -\frac{y_2}{b} - i \frac{y_1}{\rho} \right), \\ \frac{\partial}{\partial y_1} \eta^2 &= \frac{\eta^2}{2} \left( +\frac{y_1}{b} - i \frac{y_2}{\rho} \right), & \frac{\partial}{\partial y_2} \eta^2 &= \frac{\eta^2}{2} \left( +\frac{y_2}{b} + i \frac{y_1}{\rho} \right). \end{aligned} \quad (75)$$

Formulas (74) and (75) have no peculiarities over complex plane  $y_1 + iy_2$ , excluding the origin point  $0 + i0$ . From (74),(75) it follows the explicit form of derivatives with respect to direction in  $(y_1, y_2)$ -plane:

$$\begin{aligned} \nabla_{\vec{\nu}} \xi^1 &= \frac{\xi^1}{2} \left[ \frac{\rho}{a(a+a_3)} (\vec{\nu} \vec{y}) + \frac{i}{\rho} (\vec{\nu} \times \vec{y}) \right], \\ \nabla_{\vec{\nu}} \xi^2 &= \frac{\xi^2}{2} \left[ \frac{\rho}{a(a-a_3)} (\vec{\nu} \vec{y}) - \frac{i}{\rho} (\vec{\nu} \times \vec{y}) \right], \end{aligned} \quad (76)$$

and

$$\nabla_{\vec{\nu}} \eta^1 = \frac{\eta^1}{2} \left[ -\frac{\vec{\nu} \vec{y}}{b} + \frac{i}{\rho} (\vec{\nu} \times \vec{y}) \right], \quad \nabla_{\vec{\nu}} \eta^2 = \frac{\eta^2}{2} \left[ \frac{\vec{\nu} \vec{y}}{b} - \frac{i}{\rho} (\vec{\nu} \times \vec{y}) \right], \quad (77)$$

where the notation is used:

$$\vec{y} = (y_1, y_2), \quad \vec{\nu} = (\nu_1, \nu_2), \quad (\vec{\nu} \vec{y}) = \nu_1 y_1 + \nu_2 y_2, \quad (\vec{\nu} \times \vec{y}) = \nu_1 y_2 - \nu_2 y_1.$$

Relations (76) and (77) can be considered alternatively as basic equations that prescribe the explicit form of spinors  $\xi(y)$  and  $\eta(y)$  – from which we had started in the beginning. Such understanding of equations of the type (76) and (77) appears to be interesting and possibly fruitful. As for now, they do not look simple or fundamental anyhow, however having been in their infancy they do have exiting mathematical potential.

Let us examine certain interesting properties of the mapping  $(x_1, x_2, x_3) \implies \tilde{G}(y_1, y_2, y_3)$ , which look the same for both models  $\tilde{E}_3$  and  $\tilde{\Pi}_3$ . For neighborhood of any point  $\vec{y}_0$

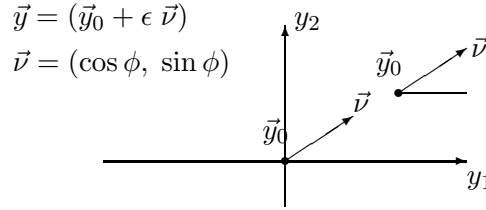


FIG. 15. Neighborhood of the point  $\vec{y}_0$

in  $x$ -representation one gets

$$\begin{aligned} x_1 &= x_1^0 + \epsilon(y_1^0 \cos \phi - y_2^0 \sin \phi) + (\epsilon^2/2) \cos 2\phi, \\ x_2 &= x_2^0 + \epsilon(y_1^0 \sin \phi + y_2^0 \cos \phi) + (\epsilon^2/2) \sin 2\phi. \end{aligned}$$

If  $y_1^0 = 0$  and  $y_2^0 = 0$ , the first order terms vanish and we have

$$x_1 = +(\epsilon^2/2) \cos 2\phi, \quad x_2 = +(\epsilon^2/2) \sin 2\phi.$$

The latter means that in the vicinity of  $(0,0)$ -point just the angle  $2\phi$  (in contrast to  $\phi$ -variable) has a first-hand geometrical sense. In accordance with  $\phi \in [0, 2\pi]$  (here an extended  $y_1, y_2$ -range has been presupposed) the variable  $2\phi$  runs through the double interval  $[0, 4\pi]$ . The part (sub-interval)  $\phi \in [0, 2\pi]$  there corresponds to the first sheet and the part  $\phi \in [2\pi, 4\pi]$  – to the second one of the 2-sheeted  $(x_1, x_2)$ -plane. In all remaining points the plane  $y_1 + y_2$ , first-hand geometrical meaning of  $\phi$ -variable follows from the formulas

$$\begin{aligned} x_1 &= a_1^0 + \epsilon \sqrt{(y_1^0)^2 + (y_2^0)^2} \cos(\phi + \Delta(y)), \\ x_2 &= a_2^0 + \epsilon \sqrt{(y_1^0)^2 + (y_2^0)^2} \sin(\phi + \Delta(y)), \end{aligned}$$

where  $\Delta(y)$  is defined by

$$\cos \Delta(y) = \frac{y_1^0}{\sqrt{(y_1^0)^2 + (y_2^0)^2}}, \quad \sin \Delta(y) = \frac{y_2^0}{\sqrt{(y_1^0)^2 + (y_2^0)^2}}.$$

This means that at all such points the variable  $\phi$  ranging in  $[0, 2\pi]$ -interval has ordinary geometrical sense.



The property just described can be reformulated as follows: all points of  $\tilde{E}_3$  and  $\tilde{\Pi}_3$ , different from  $(0, 0, x_3)$ , are characterized by  $2\pi$ -neighborhoods of directions, whereas in the vicinity of all point  $(0, 0, x_3)$  there exist  $4\pi$ -neighborhoods of directions. Evidently, that geometrical construction is well known in the complex variable function theory as it concerns 2-sheeted complex plane.

Else one point may be noticed. In all  $2\pi$ -points of the extended space spinors  $\xi(y)$  and  $\eta(y)$  are single-valued functions of spatial points  $(y_1, y_2, y_3)$ ; whereas in all  $4\pi$ -points (the whole axis  $(0, 0, x_3)$ ) spinors are not single-valued functions – they have discontinuity described by the exponential factor  $e^{\pm i\gamma/2}$ . As the variable  $\gamma$  ranges from 0 to  $4\pi$ , we will have in all  $4\pi$ -points

$$\xi(\gamma = 0) = \xi(\gamma = 4\pi) , \quad \eta(\gamma = 0) = \eta(\gamma = 4\pi) .$$

In other words, spinor  $\xi(y)$  and  $\eta(y)$  are continuous in every point of the whole space with respect to its direction set. The latter may be characterized symbolically as follows:

$$2\pi \otimes \pi \text{ for } 2\pi - \text{points} ; \quad 4\pi \otimes \pi \text{ for } 4\pi - \text{points} .$$

In the following, for the sets of discontinuity points we will employ designation  $R^{exp}$ ,  $R^{\pm 1}$  and  $\tilde{R}^{exp}$ , where symbol of tilde refers to extended models. The domains  $R^{exp}$ ,  $R^{\pm 1}$  are presented in initial vector models, in spinor models there only domain  $\tilde{R}^{exp}$  arises. The latter can be illustrated by the diagram:

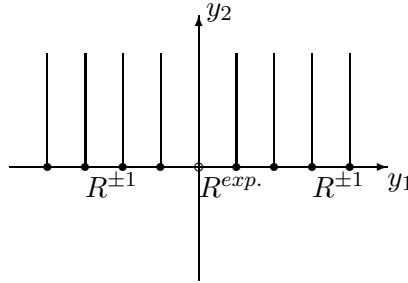


FIG.16.  $G(y_1, y_2)^{y_3}$  for  $\Pi_3$  and  $E_3$

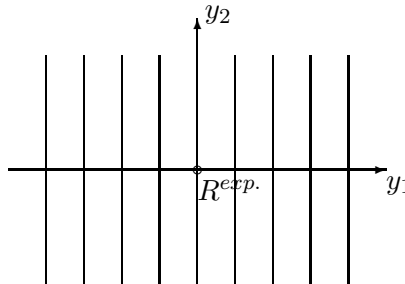


FIG. 17.  $\tilde{G}(y_1, y_2)^{y_3}$  for  $\tilde{\Pi}_3$  and  $\tilde{3}$

Manifestation of sets  $R^{exp}$  and  $\tilde{R}^{exp}$  differ from each other when  $\gamma \in [0, 2\pi]$  and  $\gamma \in [0, 4\pi]$  respectively. In view of such continuity properties of spinors  $\xi(y)$  and  $\eta(y)$  one may generalize the concept of a point of spinor space

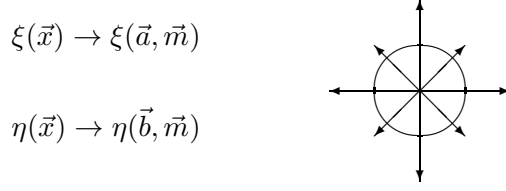


FIG. 18. Point of spinor space

that is "a point of spinor space" is an aggregate formed by the point  $\vec{x} = (y_1, y_2, y_3)$  as such and by the direction set  $\{\vec{m}\}$  near the point.

And a final remark. In the spinor space models one can readily determine a metric structure with the help of ordinary metric tensor in cylindrical parabolic coordinates

$$dl^2 = [ dy_3^2 + (y_1^2 + y_2^2)(dy_1^2 + dy_2^2) ] \quad (78)$$

where coordinates range in the extended domain, covering initial vector space twice:

$$\tilde{G}((y_1, y_2, y_3), \quad y_i \in (-\infty, +\infty), \quad i = 1, 2, 3).$$

The case of cylindrical parabolic coordinates provides us with important tool to describe spinor spaces. In a sense, the structure of spaces with spinor properties in terms of these coordinates looks simpler than of vector space – compare identification rules for boundary points. However it must be mentioned else one time: to distinguish between spinor models of different  $P$ -type, the given specification of  $\tilde{G}(y_1, y_2, y_3)$  (geometrical dimension and boundary identification) is not sufficient, and some additional mathematical technique should be elaborated.

## 12 Spinors $\xi$ and $\eta$ in parabolic coordinates

In this Section we are going to examine in spinor approach the well-known parabolic coordinates. They are defined by the formulas

$$x_1 = y_1 y_2 \cos y_3, \quad x_2 = y_1 y_2 \sin y_3, \quad x_3 = \frac{y_1^2 - y_2^2}{2}, \quad y_1, y_2 \in [0, +\infty), \quad y_3 \in [0, 2\pi]$$

with the diagram

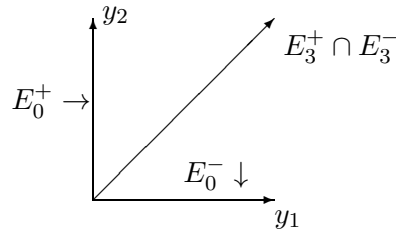


FIG. 19. Parabolic coordinates

Spatial spinor  $\eta$  of the properly vector model is given by

$$\begin{aligned}\eta^+(y) &= \frac{1}{\sqrt{2}} \begin{pmatrix} (y_1 - y_2) & e^{-iy_3/2} \\ (y_1 + y_2) & e^{+iy_3/2} \end{pmatrix}, \\ \eta^-(y) &= \frac{1}{\sqrt{2}} \begin{pmatrix} (y_2 - y_1) & (-e^{-iy_3/2}) \\ (y_2 + y_1) & e^{+iy_3/2} \end{pmatrix}.\end{aligned}\quad (79)$$

Spinors  $\eta_0^\pm, \eta^{+\cap-}$  look as follows

$$\eta_0^+ = \frac{y_1}{\sqrt{2}} \begin{pmatrix} e^{-i\Gamma/2} \\ e^{+i\Gamma/2} \end{pmatrix}, \quad \eta_0^- = \frac{y_1}{\sqrt{2}} \begin{pmatrix} -e^{-i\Gamma/2} \\ e^{+i\Gamma/2} \end{pmatrix}, \quad \eta^{+\cap-} = \begin{pmatrix} 0 \\ \sqrt{2}ye^{+iy_3/2} \end{pmatrix}.$$

where  $\Gamma$  is a mute variable, the notation  $y_1 = y_2 = y$  is used for the plane  $x_3 = 0$ .

As for pseudo vector model  $\Pi_3$  we will have

$$\xi(y) = \begin{pmatrix} y_1 e^{-iy_3/2} \\ y_2 e^{+iy_3/2} \end{pmatrix}. \quad (80)$$

On comparing (80) with definition of spatial spinor

$$\xi(y) = \begin{pmatrix} N e^{-i\gamma/2} \\ M e^{+i\gamma/2} \end{pmatrix}.$$

we immediately arrive at

$$y_1 = N, \quad y_2 = M, \quad y_3 = \gamma. \quad (81)$$

In other words, parabolic coordinates  $(y_1, y_2, y_3)$  just coincide with  $(N, M, \gamma)$  introduced in Sec. 1 at defining the concept of spinor  $\eta$ .

Now let us outline some details of continuity property of spinors  $\xi$  and  $\eta$ .

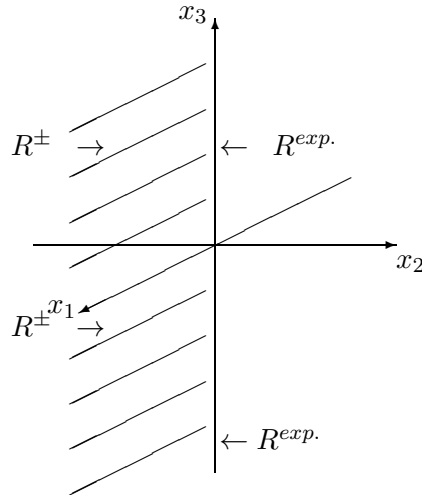


FIG. 20.  $R^{\pm 1}, R^{exp}$  in  $x$ -representation

and

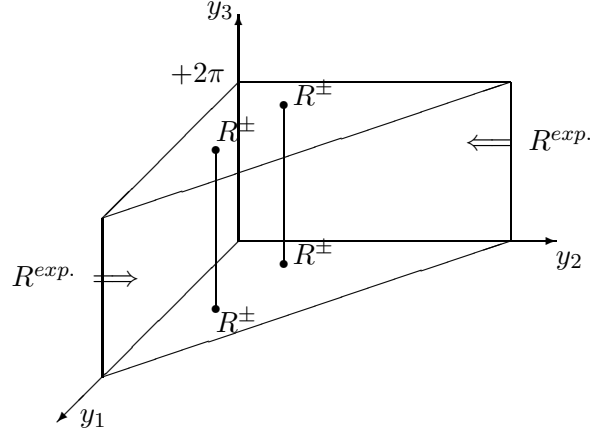


FIG 21.  $R^{\pm 1}, R^{exp}$  in  $y$ -representation

Transition to extended space is achieved by doubling the above domain  $G(y) \Rightarrow \tilde{G}(y)$

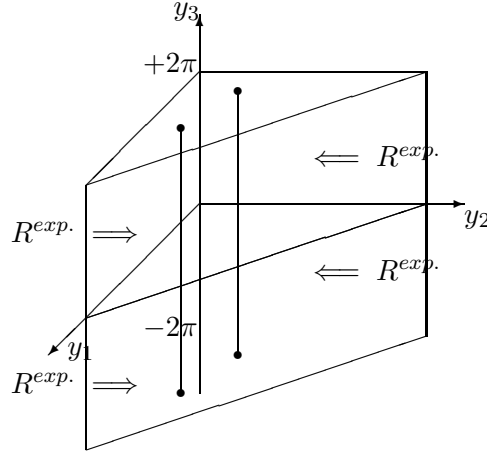


FIG. 22. Region  $\tilde{G}(y)$

In parabolic coordinates spatial metrics is

$$dl^2 = [ (y_1^2 + y_2^2) (dy_1^2 + dy_2^2) + y_1^2 y_2^2 dy_3^2 ]$$

or

$$dl^2 = (M^2 + N^2) (dM^2 + dN^2) + M^2 N^2 d\gamma^2 .$$

One final remark in this Section. You do not need to employ necessarily the domain  $\tilde{G}(y)$  described above

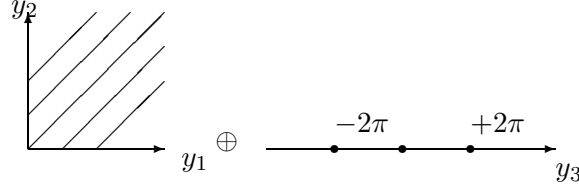


FIG. 23.  $\tilde{G}(y)$

where the key role in extending procedure is assigned to angle variable  $y_3 = \gamma$ . Alternatively, instead another (alternative, simple and symmetrical) possibility exists

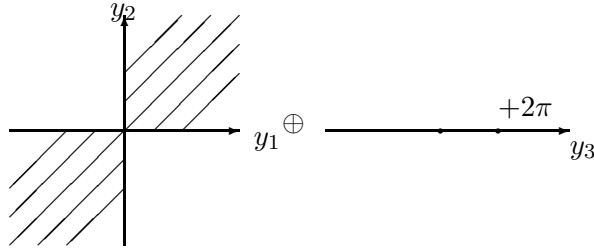


FIG. 24.  $\tilde{G}(y)$

Thus, various domains  $\tilde{G}(y)$  are acceptable for correct parameterizations of spinor spaces, and you may choose any for reason of convention.

### 13 Connection between $\xi$ and $\eta$ models

Two models of spinors spaces with respect to  $P$ -orientation are grounded on different mappings  $\xi$  and  $\eta$  defined over the same extended domain  $\tilde{G}(y_i)$ . The natural question is: how are these two maps connected to each others. An answer can be found on comparing the formulas for  $\xi$  and  $\eta$ :

$$\eta(y) = \frac{1}{\sqrt{2}} \begin{pmatrix} (y_1 - y_2) e^{-iy_3/2} \\ (y_1 + y_2) e^{+iy_3/2} \end{pmatrix}, \quad \xi(y) = \begin{pmatrix} y_1 e^{-iy_3/2} \\ y_2 e^{+iy_3/2} \end{pmatrix}. \quad (82)$$

From (82) we immediately arrive at

$$\eta_1 = \frac{\xi_1 - \xi_2^*}{\sqrt{2}}, \eta_2 = \frac{\xi_1^* + \xi_2}{\sqrt{2}}, \quad \eta = \frac{1}{\sqrt{2}}(\xi - i \sigma^2 \xi^*). \quad (83)$$

Inverse to (83) looks as

$$\xi_1 = \frac{\eta_1 + \eta_2^*}{\sqrt{2}}, \quad \xi_2 = \frac{\eta_2 - \eta_1^*}{\sqrt{2}}, \quad \xi = \frac{1}{\sqrt{2}} (\eta - i \sigma^2 \eta^*). \quad (84)$$

In fact, the formulas (83) and (84) are not coordinate-dependent – one may obtain them with the use of any other coordinate system. As for (83) and (84) there are two points that deserve special attention:

1) complex conjugation enters them explicitly which correlates with the change in orientation properties of the models;

2) spinors  $\xi$  and  $i\sigma^2\xi^*$  (as well as  $\eta$  and  $i\sigma^2\eta^*$ ) provide us with non-equivalent representations of the extended unitary group  $\tilde{SU}(2)$ .

## 14 Spatial spinors in spherical coordinates

In this Section we will examine in spinor approach the most commonly encountered system of spherical coordinates. These are defined by

$$x_1 = y_1 \sin y_2 \cos y_3, \quad x_2 = y_1 \sin y_2 \sin y_3, \quad x_3 = y_1 \cos y_2, \\ y_1 \in [0, +\infty), \quad y_2 \in [0, +\pi], \quad y_3 \in [0, +2\pi]. \quad (85)$$

Spinor  $\eta(y)$  of pseudo vector model  $\Pi_3$  is given by

$$\xi = \begin{pmatrix} \sqrt{y_1(1 + \cos y_2)} e^{-iy_3/2} \\ \sqrt{y_1(1 - \cos y_2)} e^{+iy_3/2} \end{pmatrix}, \quad \xi^{+\cap-} = \sqrt{y_1} \begin{pmatrix} e^{-iy_3/2} \\ e^{+iy_3/2} \end{pmatrix}, \quad (86)$$

$$\xi_0^+ = \sqrt{2y_2} \begin{pmatrix} e^{-i\Gamma/2} \\ 0 \end{pmatrix}, \quad \xi_0^- = \sqrt{2y_1} \begin{pmatrix} 0 \\ e^{+i\Gamma/2} \end{pmatrix}, \quad (\Gamma = y_3). \quad (87)$$

In turn, spinor  $\eta(y)$  of properly vector model  $E_3$  is defined according to

$$\eta = \begin{pmatrix} \sqrt{y_1(1 - \sin y_2)} (\sigma e^{-iy_3/2}) \\ \sqrt{y_1(1 + \sin y_2)} e^{+iy_3/2} \end{pmatrix}, \\ \eta^{+\cap-} = \sqrt{y_1} \begin{pmatrix} 0 \\ \sqrt{2y_1} e^{+iy_3/2} \end{pmatrix}, \quad \eta_0^+ = \begin{pmatrix} e^{-i\Gamma/2} \\ e^{+i\Gamma/2} \end{pmatrix}; \quad \eta_0^- = \begin{pmatrix} -e^{-i\Gamma/2} \\ e^{+i\Gamma/2} \end{pmatrix}. \quad (88)$$

Discontinuity properties of these spinors may be characterized by the diagram

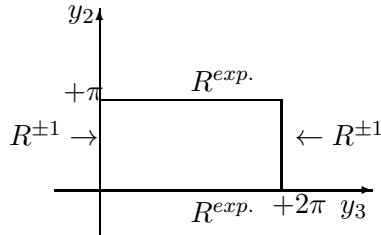


FIG. 25. Spinor discontinuity in spherical coordinates

Evidently, transition to extended models can be realized through formal doubling the range of angle variable  $y_3$  ( in the following we will use the more common notation  $y_1 = r, y_2 = \theta, y_3 = \phi$  )

$$\tilde{G}(r, \theta, \phi) = \{ r \in [0, +\infty) , \theta \in [0, +\pi], \phi \in [-2\pi, +2\pi] \} . \quad (89)$$

Now let us discuss some alternative variants of extended domain  $\tilde{G}$  that might be used for covering spinor spaces. By way of illustration, the most natural and symmetrical possibility of this type is to extend the range of radial variable:

$$\tilde{G}'(r, \theta, \phi) = \{ r \in (-\infty, +\infty) , \theta \in [0, +\pi], \phi \in [-\pi, -\pi] \} . \quad (90)$$

To prove it, let us turn again to the above expression for  $\xi$

$$\xi(r, \theta, \phi) = \begin{pmatrix} \sqrt{1 + \cos \theta} & (\sqrt{r} e^{i\phi})^* \\ \sqrt{1 - \cos \theta} & (\sqrt{r} e^{i\phi}) \end{pmatrix} . \quad (91)$$

This function is considered over the old domain  $\tilde{G}(r, \phi, \theta)$ :

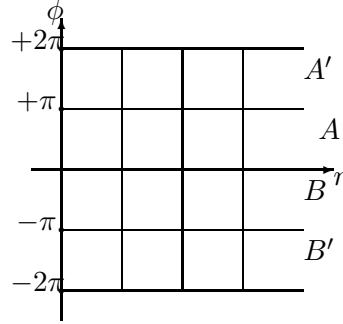


FIG. 26.  $(\phi, r)$  region of spinor space

where vertical lines have joined identified points of the boundary set. Taking into account the equality  $r e^{\pm(\phi \pm \pi)} = (-r) e^{\pm i\phi}$ , and allowing for connection between  $(A, A')$  and  $(B, B')$  sub-sets

$$\phi^{A'} = (\phi^A + \pi) , \quad \phi^{B'} = (\phi^B - \pi) ,$$

we readily arrive at two relations

$$\xi(r, \theta, \phi^{A'}) = \xi(-r, \theta, \phi^A) , \quad \xi(r, \theta, \phi^{B'}) = \xi(-r, \theta, \phi^B) , \quad (92)$$

which provide us with possibility to employ the following  $(r, \phi)$ -domain:

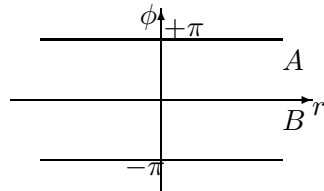


FIG. 27. Alternative  $(\phi, r)$  region of spinor space

Observing identified points on the initial diagram ( $L \equiv L', F \equiv F'$  and so on )

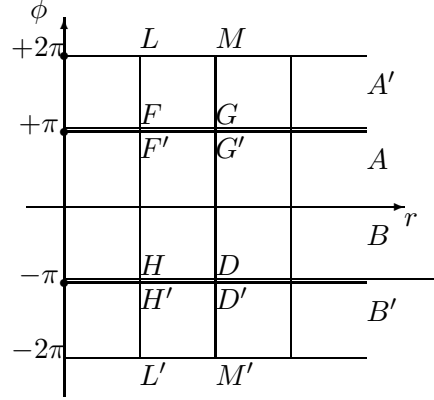


FIG. 28. Identification in  $(\phi, r)$  region

you can easily derive identification rules on the new diagram for  $\tilde{G}'$ :

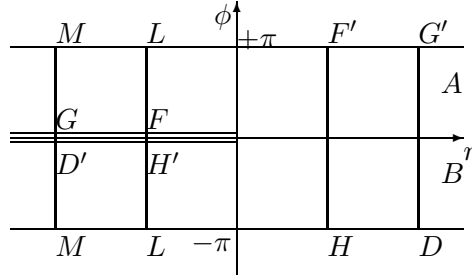


FIG. 29. Identification in alternative  $(\phi, r)$  region

Transformation of the domain  $\tilde{G}$  into  $\tilde{G}'$  can be illustrated by the symbolic relation

$$[R^+ \times (A' + A + B + B')] \sim [(R^+ + R^-) \times (A + B)]. \quad (93)$$

Needless to say that two domains  $\tilde{G}$  and  $\tilde{G}'$  just indicated are not the only possible. For example, taking into account identities

$$r e^{\pm i(\phi \pm \pi)} = (-r) e^{\pm i\phi}, \quad r e^{\pm i(\phi \pm 3\pi)} = (-r) e^{\pm i\phi},$$

and relationships between  $(A, B)$  and  $(A', B')$

$$\phi^B = (\phi^A - \pi), \quad \phi^{B'} = (\phi^{A'} - 3\pi)$$

we readily produce the formulas

$$\xi(r, \theta, \phi^B) = \xi(-r, \theta, \phi^A), \quad \xi(r, \theta, \phi^{B'}) = \xi(-r, \theta, \phi^{A'}). \quad (94)$$



They mean that instead of

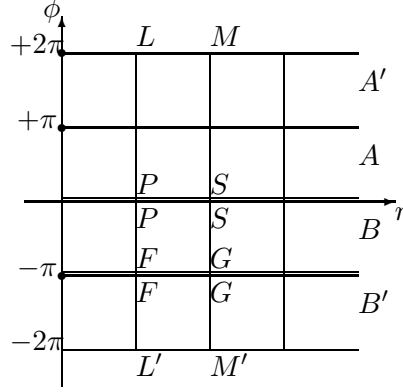


FIG. 30. Else one transformation of  $(\phi, r)$  region

you may use yet another set  $\tilde{G}''$

$$\bar{G}''(r, \theta, \phi) = \{r \in (-\infty, +\infty), \theta \in [0, +\pi], \phi \in [0, +2\pi]\};$$

$$[R^+ \times (A' + A + B + B')] \sim [(R^+ + R^-) \times (A + A')]$$

with identification rules as follows

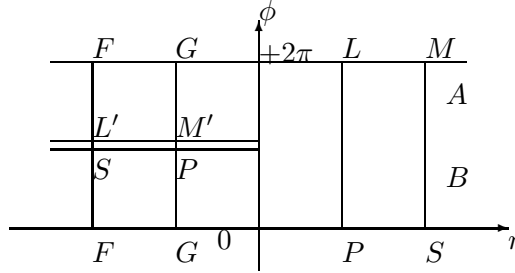


FIG. 31. Yet else one  $(\phi, r)$  region

It is self-evident that everything said about  $\xi$ -model is suitable for another spinor model  $\eta$  as well.

## 15 Conclusion

The results obtained for 3-space with  $(x, y, z)$  coordinates should be extended to Minkowski 4-space with coordinates  $(t, x, y, z)$ . Mathematically it means the use of relativistic  $SL(2, C)$  spinors instead of non-relativistic  $SU(2)$  spinors.

Domains of curvilinear coordinates associated with spinor space can be used to examine possible quantum mechanical manifestation of the spinor structure both in non-relativistic and relativistic theories. To this end, one should specially look at analytical properties of the known solutions of the Schrödinger and Dirac equations in various coordinates.

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